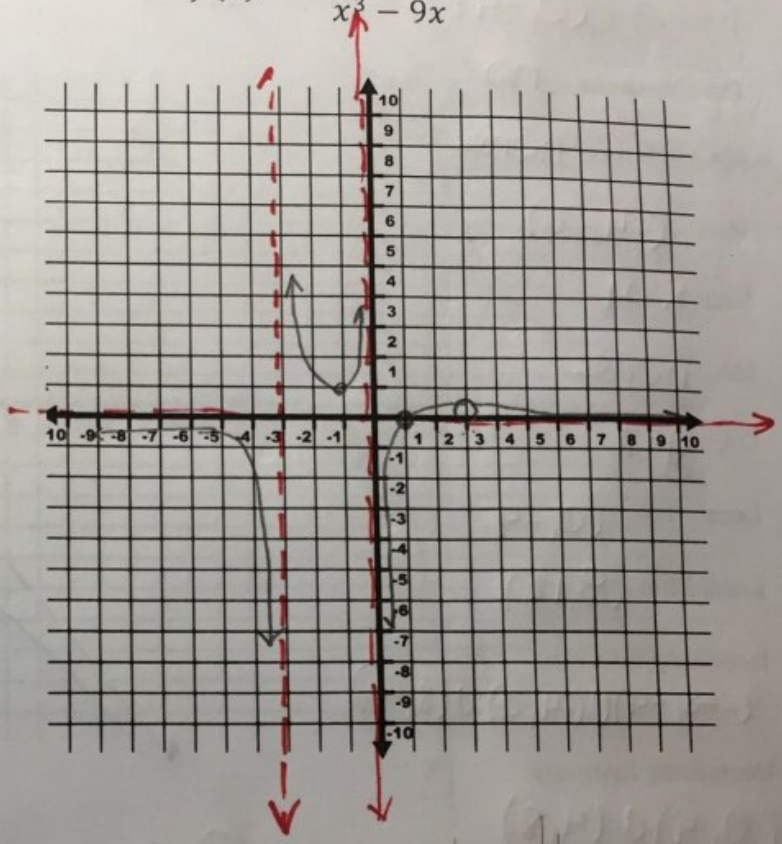
Name Key

Analyzing Rational Graphs

$$f(x) = \frac{x^2 - 4x + 3}{x^3 - 9x}$$



x-intercepts: (1,0)

y-intercepts: ~~3~~ none

OA-intercepts: none

HA-intercepts: $x = 1$

Hole: $(3, 1/9)$

VA: $x = 0$ $x = -3$

HA: $y = 0$

OA: none

Local Max: (3, hole)

Local Min: (-1, 1)

Increasing Intervals:

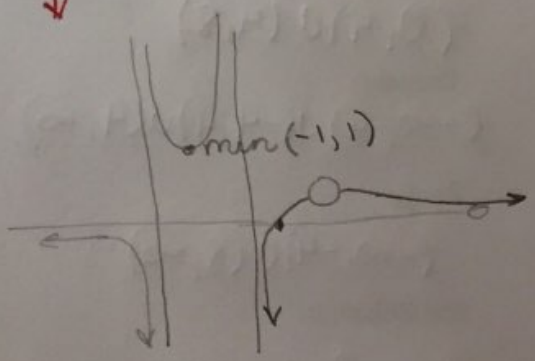
$(-1, 0) \cup (0, 3)$

Decreasing Intervals:

$(-\infty, -3) \cup (-3, -1) \cup (3, \infty)$

Domain: $(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$

Range: $(-\infty, 1/9) \cup (1, \infty)$



End Behavior:

$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = 0$

HA.-intercepts

HA → $0 = \frac{x^2 - 4x + 3}{x^3 - 9x}$ ← Eq

$0 = (x-3)(x-1)$

~~$x = 3$~~ $x = 1$

hole

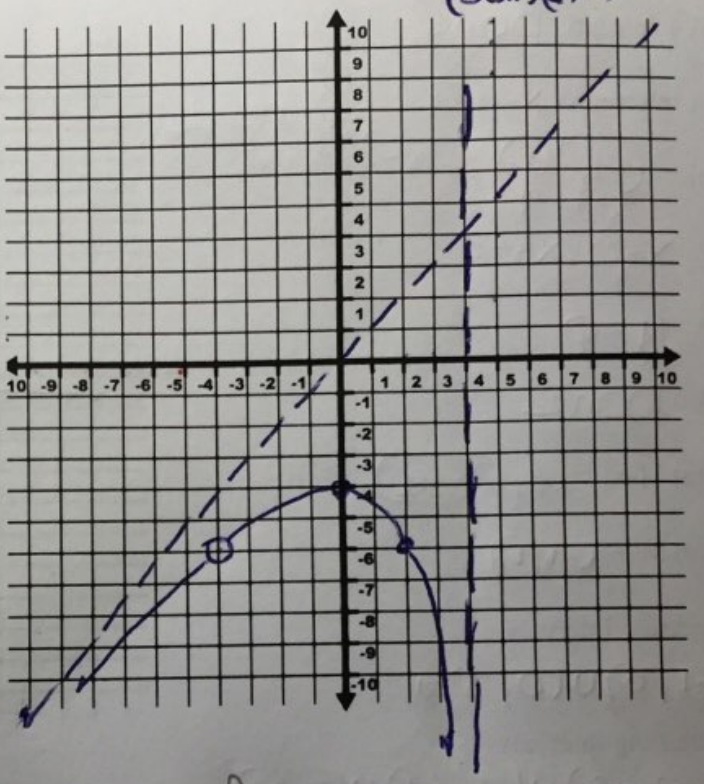
Original	factor	reduce
• H.A: $y = 0$	$\frac{(x-3)(x-1)}{x(x+3)(x-3)}$	• hole $(3, 1/9)$
• y-int none		• x-int (1,0)
• O.A: none	• Domain	• VA $x = 0$, $x = -3$
		• HA $y = 0$

$\frac{x-1}{x(x+3)}$

Domain

$(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$

$f(x) = \dfrac{x^3 + 64}{x^2 - 16}$



x-intercepts: none

y-intercepts: $(0,-4)$

OA-intercepts: no

HA-intercepts: none

Hole: $(-4,-6)$

VA: $x=4$

HA: none

OA: $y=x$

Local Max: $(0,-4$

Local Min: $(8,12)$

Increasing Intervals:
$(-\infty,-4) \cup (-4,0) \cup (8,\infty)$

Decreasing Intervals:
$(0,4) \cup (4,8)$

Domain:
$(-\infty,-4) \cup (-4,4) \cup (4,\infty)$

Range:
$(-\infty,-4) \cup (8,\infty)$

End Behavior:

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

$x = \dfrac{x^2-4x+16}{x-4}$

$x^2-4x = x^2-4x+16$

$0 = 16$

Orig
H.A none
y-int $(0,-4)$

factor
$\dfrac{(x+4)(x^2-4x+16)}{(x+4)(x-4)}$

O.A

$x^2-16 \overline{)\, x^3+0x^2+x+64}$ $\quad x$

$x^3 \quad -16$

$y = x$

hole $(-4,6)$
$x=-4$
$\dfrac{(-4)^2-4(-4)+16}{-4-4} = -6$

Domain

$-4 \quad 4$

$(-\infty,-4) \cup (-4,4) \cup (4,\infty)$

x-intercepts: $(-4,0)$

y-intercepts: $(0,-2)$

OA-intercepts: none

HA-intercepts: $1 = \frac{x+4}{x-2}$ none

$x-2 = x+4$

Hole: $(-1,-1)$

VA: $x=2$

HA: $y=1$

OA: none

Local Max: none

Local Min: none

Increasing Intervals:

$(2,\infty)$

Decreasing Intervals:

$(-\infty,-1) \cup (-1,2)$

Domain: -1 2

$(-\infty,-1) \cup (-1,2) \cup (2,\infty)$

Range:

$(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

End Behavior:

$\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to \infty} f(x) = 1$

$$f(x) = \frac{x^2+5x+4}{x^2-x-2} = \frac{(x+4)(x+1)}{(x-2)(x+1)}$$