PreCalculus
Unit 5: Analytic Trig Review

Name Key

Date _____

Solve each equation.

1. $4\cos^2 x - 3 = 0$

$x = \pi/6 \pm 2\pi k \quad x = \frac{11\pi}{6} \pm 2\pi k$

$x = 5\pi/6 \pm 2\pi k \quad x = \frac{7\pi}{6} \pm 2\pi k$

2. $2\sin^2 x - \sin x - 3 = 0, \quad 0 \le x < 2\pi$

$x = \frac{3\pi}{2}$

3. $\sin(3\theta) = -1, \quad 0 \le \theta < 2\pi$

$\theta = \pi/2 \; ; \; 7\pi/6 \; ; \; 11\pi/6$

4. $\sqrt{3} + \tan(2\theta) = 0 \quad 0 \le x < 2\pi$

$\theta = \pi/3 \; ; \; 5\pi/6 \; ; \; 8\pi/6 \; ; \; 11\pi/6$

5. $\sin\left(\frac{\theta}{2}\right) - 1 = 0$

$\theta = \pi + 4\pi k$

6. $\sec\left(x - \frac{\pi}{5}\right) = 2, \quad 0 \le x < 2\pi$

$\theta = \frac{8\pi}{15} \quad \frac{28\pi}{15}$

7. $4\cos^2 x + 4\cos x - 3 = 0, \quad 0 \le x < 2\pi$

$x = \pi/3 \; ; \; 5\pi/3$

8. $2\tan^2 x - \tan x - 6 = 0, \quad 0 \le x < 2\pi$

$x = 2.159, 5.300$

$x = 1.107, 4.249$

9. $\sin\theta = -0.2, \quad 0 \le \theta < 2\pi$

$\theta = 6.082 \; ; 3.343$

10. $\tan x = 5, \quad 0 \le x < 2\pi$

$x = 1.373, 4.515$

Determine the exact value of each trigonometric function using a sum/difference and then again using a half/double angle to verify.

11. $\cos\frac{7\pi}{12}$

Cos(135-30)

$\frac{-\sqrt{6}+\sqrt{2}}{4}$

half ∠

$\frac{-\sqrt{2-\sqrt{3}}}{2}$

12. $\tan\left(-\frac{\pi}{12}\right)$

Sum/diff

$-2+\sqrt{3}$

half/double

$-2+\sqrt{3}$

13. $\sin\frac{5\pi}{12}$

sum/diff

$\frac{\sqrt{6}+\sqrt{2}}{4}$

half/double

$\frac{\sqrt{2+\sqrt{3}}}{2}$

Prove each trigonometric identity.

14. $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} - \sin^2\theta =$

$1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$

15. $4\cos^2\theta + 3\sin^2\theta = 3 + \cos^2\theta$

$4\cos^2\theta + 3(1-\cos^2\theta)$

$4\cos^2\theta + 3 - 3\cos^2\theta$

$\cos^2\theta + 3 = 3 + \cos^2\theta$

16. $\frac{\csc\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\cos^2\theta}$

$\frac{\frac{1}{\sin\theta}}{1+\frac{1}{\sin\theta}}$

$= \frac{\frac{1}{\sin\theta}}{\frac{\sin\theta+1}{\sin\theta}}$

$= \frac{1}{\sin\theta+1}$

$\frac{1}{\sin\theta+1}$

$= \frac{1}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$

$\frac{1-\sin\theta}{1-\sin^2\theta}$

$= \frac{1-\sin\theta}{\cos^2\theta} =$

17. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2$

$= \tan^2\theta(\tan^2\theta+1)$

$= \tan^2\theta(\sec^2\theta)$

$= (\sec^2\theta - 1)(\sec^2\theta)$

$= \sec^4\theta - \sec^2\theta$

18. $\frac{\sin\theta+\cos\theta}{\sin\theta} - \frac{\cos\theta-\sin\theta}{\cos\theta} = \sec\theta\csc\theta$

$\frac{\sin\theta\cos\theta + \cos^2\theta - \sin\theta\cos\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}$

$= \sec\theta\cdot\csc\theta = \sec\theta\csc\theta$

19. $\sin(\alpha-\beta)\sin(\alpha+\beta) = \sin^2\alpha - \sin^2\beta$

$(\sin\alpha\cos\beta - \cos\alpha\sin\beta)(\sin\alpha\cos\beta + \cos\alpha\sin\beta) =$

$\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$

$\sin^2\alpha(1-\sin^2\beta) - (1-\sin^2\alpha)\sin^2\beta$

$\sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta$

$\sin^2\alpha - \sin^2\beta = \sin^2\alpha - \sin^2\beta$

20. $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$2\cos\alpha\cos\beta = 2\cos\alpha\cos\beta$

21. $\frac{\cos(2\theta)}{1+\sin(2\theta)} = \frac{\cot\theta-1}{\cot\theta+1}$

$= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\frac{\cos\theta-\sin\theta}{\sin\theta}}{\frac{\cos\theta+\sin\theta}{\sin\theta}}$

$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\left(\frac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta}\right)$

$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}$

$\frac{\cos(2\theta)}{1+\sin(2\theta)}$

22. $\frac{1+\sin\theta}{1-\sin\theta}$

22. $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$

$1+2\sin\theta+\sin^2\theta - 1+2\sin\theta-\sin^2\theta$

$\frac{4\sin\theta}{1-\sin^2\theta}$

$\frac{4\sin\theta}{\cos^2\theta} = \frac{4\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} = 4\tan\theta\sec\theta$

23. $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

$\frac{1-\sin x}{\cos x}\left(\frac{1+\sin x}{1+\sin x}\right)$

$\frac{1-\sin^2 x}{\cos x(1+\sin x)}$

$\frac{\cos^2 x}{\cos x(1+\sin x} = \frac{\cos x}{1+\sin x}$

24. $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

omit

25. $\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta} = \cos(2\theta)$

$\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$

$\frac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}$

$\frac{\cos^2\theta-\sin^2\theta}{1}$

$= \cos(2\theta)$

26. $\tan\frac{\theta}{2} = \csc\theta - \cot\theta$

$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$\frac{1-\cos\theta}{\sin\theta}$

$\tan\frac{\theta}{2}$

27. $\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

$\sin\left(\frac{3\pi}{2}\right)\cos\theta + \cos\frac{3\pi}{2}\sin\theta$

$-1\cos\theta + 0\sin\theta$

$= -1\cos\theta$

# Review of Law of Sines & Cosines

Date_____

**Find each measurement indicated. Round your answers to the nearest tenth.**

1) $a = 12$ cm, $b = 21$ cm, $c = 13$ cm
Find $m\angle A$

$m\angle A \approx 31.4°$

2) $m\angle A = 113°$, $b = 19$ mi, $c = 17$ mi
Find $m\angle B$

$m\angle B = 35.6°$

3) $m\angle A = 52°$, $c = 35$ m, $a = 30$ m
Find $m\angle C$

$\angle C = 66.8°$
$\angle C \approx 113.2°$

4) $m\angle C = 143°$, $b = 6$ in, $c = 26$ in
Find $m\angle B$

$m\angle B = 8°$

5) $c = 7$ cm, $m\angle B = 30°$, $a = 19$ cm
Find $b$

$b = 13.4$cm

6) $a = 12$ in, $c = 29$ in, $m\angle B = 51°$
Find $b$

$b = 23.4$cm

7) $m\angle A = 18°$, $c = 34$ km, $a = 23$ km
Find $b$

$b = 52.8$ km
$b \approx 11.9$ km

8) $m\angle B = 57°$, $a = 35$ mi, $b = 20$ mi
Find $c$

not a △