

Name: Key

Trig Equation Practice

Solve each equation on $0 \leq \theta \leq 2\pi$. Give exact answers in terms of π .

1.) $2\sin\theta + 3 = 2$ $2\sin\theta = -1$
 $\sin\theta = -1/2$
 $\theta = \sin^{-1}(-1/2)$

$\theta = 7\pi/6 ; \theta = 11\pi/6$

2.) $2\sin^2\theta - 1 = 0$ $\sin^2\theta = 1/2$
 $\sin\theta = \pm \sqrt{1/2}$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

3.) $4\cos^2\theta - 3 = 0$ $\cos^2\theta = 3/4$
 $\cos\theta = \pm \sqrt{3}/2$

$\theta = \pi/6, 11\pi/6, 5\pi/6, 7\pi/6$

4.) $\tan\theta + 1 = 0$ $\tan\theta = -1$

$\theta = 3\pi/4, 7\pi/4$

5.) $\sqrt{3}\cot\theta + 1 = 0$ $\cot\theta = -1/\sqrt{3}$
 $\tan\theta = -\sqrt{3}$

$\theta = 2\pi/3, 5\pi/3$

6.) $4\sec\theta + 6 = -2$ $\sec\theta = -2$
 $\cos\theta = -1/2$

$\theta = 2\pi/3, 4\pi/3$

7.) $2\cos^2\theta + \cos\theta = 0$ $\cos\theta = 0$ $\left\{ \begin{array}{l} 2\cos\theta + 1 = 0 \\ \cos\theta = -1/2 \end{array} \right.$

$\theta = \pi/2, 3\pi/2, 2\pi/3, 4\pi/3$

GCF $\cos\theta(2\cos\theta + 1) = 0$

8.) $2\sin^2\theta - \sin\theta - 1 = 0$

$2\sin\theta + 1 = 0$ $\left\{ \begin{array}{l} \sin\theta - 1 = 0 \\ \sin\theta = -1/2 \end{array} \right.$ $\sin\theta = 1$

$\theta = \pi/2, 7\pi/6, 11\pi/6$

* think $2x^2 - x - 1 = 0$

* factor $(2\sin\theta + 1)(\sin\theta - 1) = 0$

9.) $(\tan\theta - 1)(\sec\theta - 1) = 0$ hint: set each factor = 0 and solve

$\tan\theta = 1$ $\sec\theta = 1$
 $\theta = \pi/4, 5\pi/4, 0$

10.) $1 - \cos^2\theta = 1 + \cos\theta + \cos^2\theta$

$\cos\theta = 0$ $\left\{ \begin{array}{l} 1 + 2\cos\theta = 0 \\ \cos\theta = -1/2 \end{array} \right.$

$\theta = \pi/2, 3\pi/2, 2\pi/3, 4\pi/3$

set = 0

$0 = \cos\theta + 2\cos^2\theta$

GCF $0 = \cos\theta(1 + 2\cos\theta)$

$$\begin{cases} 2\sin\theta + 1 = 0 \\ \sin\theta = -1/2 \end{cases} \begin{cases} \sin\theta - 1 = 0 \\ \sin\theta = 1 \end{cases}$$

$$\theta = \pi/2, 7\pi/6, 11\pi/6$$

11.) $1 - \sin^2\theta + \sin\theta = \sin^2\theta$

set = 0 $0 = 2\sin^2\theta - \sin\theta - 1$

factor $0 = (2\sin\theta + 1)(\sin\theta - 1)$

$$\begin{cases} \cos\theta + 5 = 0 \\ \cos\theta = -5 \end{cases} \begin{cases} \cos\theta + 1 = 0 \\ \cos\theta = -1 \end{cases}$$

$$\theta = \pi$$

12.) $1 - \cos^2\theta = 6(\cos\theta + 1)$

$$0 = \cos^2\theta + 6\cos\theta + 5$$

$$0 = (\cos\theta + 5)(\cos\theta + 1)$$

13.) $2(1 - \cos^2\theta) = 3(1 - \cos\theta)$

$$0 = 2\cos^2\theta - 3\cos\theta + 1$$

$$0 = (\cos\theta - 1)(2\cos\theta - 1)$$

$$\begin{cases} \cos\theta - 1 = 0 \\ \cos\theta = 1 \end{cases} \begin{cases} 2\cos\theta - 1 = 0 \\ \cos\theta = 1/2 \end{cases}$$

$$\theta = 0, \pi/3, 5\pi/3$$

14.) $\sin\theta = \frac{1}{\sin\theta}$

$$\sin\theta = \pm 1$$

$$\sin^2\theta = 1$$

$$\theta = \pi/2, 3\pi/2$$

15.) $\sec^2\theta - 1 = \frac{3}{2}\sec\theta$

$$\begin{cases} \sec\theta = -1/2 \\ \sec\theta = 2 \end{cases}$$

$$\theta = \pi/3, 5\pi/3$$

$$\begin{aligned} 2\sec^2\theta - 3\sec\theta - 2 &= 0 \\ (2\sec\theta + 1)(\sec\theta - 2) &= 0 \end{aligned}$$

$$\begin{cases} \sec\theta = -2 \\ \sec\theta = 1/2 \end{cases}$$

16.) $\cos^3\theta = \cos\theta$

$$\cos^3\theta - \cos\theta = 0$$

$$\cos\theta(\cos^2\theta - 1) = 0$$

$$\begin{aligned} \cos\theta(\cos\theta + 1)(\cos\theta - 1) &= 0 \\ \cos\theta = 0 \quad \cos\theta = -1 \quad \cos\theta &= 1 \end{aligned}$$

$$\theta = \pi/2, 3\pi/2, \pi, 0$$

17.) $2\sin(2x) - \sqrt{3} = 0$

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

$$\begin{cases} 2x = \pi/3 + 2\pi k \\ x = \pi/6 + \pi k \end{cases}$$

$$\begin{cases} 2x = 2\pi/3 + 2\pi k \\ x = \pi/3 + \pi k \end{cases}$$

$$x = \pi/6, 7\pi/6, \pi/3, 4\pi/3$$

18.) $2\cos(3x) + 1 = 0$

$$\cos 3x = -1/2$$

$$\begin{cases} 3x = 2\pi/3 + 2\pi k \\ x = 2\pi/9 + 2/3\pi k \end{cases}$$

$$\begin{cases} 3x = 4\pi/3 + 2\pi k \\ x = 4\pi/9 + 2/3\pi k \end{cases}$$

$$x = 2\pi/9, 8\pi/9, 14\pi/9, 4\pi/9, 10\pi/9, 16\pi/9$$

19.) $\sin(x/3) = \frac{1}{2}$

$$\frac{x}{3} = \pi/6 + 2\pi k \quad \left\{ \quad \frac{x}{3} = \frac{5\pi}{6} + 2\pi k \right.$$

$$x = \pi/2 + 6\pi k$$

$$x = \frac{5\pi}{2} + 6\pi k$$

$$x = \pi/2$$

too big $0 \leq x < 2\pi$

20.) $1 = \sin(2x + \pi)$

$$2x + \pi = \pi/2 + 2\pi k$$

$$2x = -\pi/2 + 2\pi k$$

$$x = -\pi/4 + \pi k$$

$$x = 3\pi/4, 7\pi/4$$

21.) $\frac{-\sqrt{3}}{2} = \cos(2x)$

$$2x = \frac{5\pi}{6} + 2\pi k$$

$$2x = \frac{7\pi}{6} + 2\pi k$$

$$x = \frac{5\pi}{12} + \pi k$$

$$x = \frac{7\pi}{12} + \pi k$$

$$x = \frac{5\pi}{12}, \frac{17\pi}{12} \quad x = \frac{7\pi}{12}, \frac{19\pi}{12}$$