Extra Practice

1.) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2\csc x$

$\frac{(1+\cos x)(1+\cos x)+\sin^2 x}{\sin x(1+\cos x)} =$ ↓

$\frac{1+2\cos x+\overbrace{\cos^2 x+\sin^2 x}^{1}}{\sin x(1+\cos x)} =$ ↓

$\frac{1+2\cos x+1}{\sin x(1+\cos x)} =$ ↓

$\frac{2+2\cos x}{\sin x(1+\cos x} =$ ↓

$\frac{2(1+\cos x)}{\sin x(1+\cos x)} =$

$\frac{2}{\sin x} = 2\csc x = 2\csc x$

3.) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

$\frac{1+\cos x+1-\cos x}{1-\cos^2 x} =$ ↓

$\frac{2}{\sin^2 x} = 2\csc^2 x = 2\csc^2 x$

2.) $\frac{\csc^2 x-1}{\csc^2 x} = \cos^2 x$

$\frac{\cot^2 x}{\csc^2 x} =$ ↓

$\frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} =$ ↓

$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1} = \cos^2 x = \cos^2 x$

4) $(\cot^2 x+1)(\sin^2 x-1) = -\cot^2 x$

$(\csc^2 x)(-\cos^2 x) =$ ↓

$\frac{1}{\sin^2 x}(-\cos^2 x) =$ ↓

$-\cot^2 x = -\cot^2 x$

Sum + difference

5.) $\sin 15° = \sin(45° - 30°)$

$\sin 45° \cos 30° - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

7) $\sin 75° = \sin(45° + 30°)$

$\sin 45° \cos 30° + \cos 45 \sin 30$

$\sqrt{2}/2 \ \sqrt{3}/2 + \sqrt{2}/2 \ \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

8) $\cos 75° = \cos(45° + 30°)$

$\cos 45° \cos 30° - \sin 45° \sin 30$

$\sqrt{2}/2 \ \sqrt{3}/2 - \sqrt{2}/2 \ 1/2$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

6.) $\tan 15° = \tan(45° - 30°)$

$$\frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

$\frac{3-\sqrt{3}}{3} \cdot \frac{3}{3+\sqrt{3}} \left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$

$= \frac{9 - 3\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6}$

$= 2 - \sqrt{3}$

9) $\cos^{15°} \pi/12 = \cos(^{45° - 30°}\pi/4 - \pi/6)$

$\cos \pi/4 \cos \pi/6 + \sin \pi/4 \sin \pi/6$

$\sqrt{2}/2 \ \sqrt{3}/2 + \sqrt{2}/2 \ 1/2$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

10) $\sin 7\pi/12 = \sin(60 + 45°)$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

11) $\tan 5\pi/12 = \tan(45° + 30°)$

$= 2 + \sqrt{3}$

12) $\tan 11\pi/12 = \tan(120° + 45°)$

$-2 + \sqrt{3}$

13) $\sin(-\pi/12)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

14) $\cos 7\pi/12 = \frac{\sqrt{2}-\sqrt{6}}{4}$

15.) $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$

$\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta=$ ↓

$2\sin\alpha\cos\beta=2\sin\alpha\cos\beta$

16.) $\dfrac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta}=1+\cot\alpha\tan\beta$

↓

$\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta}=$ ↓

$\dfrac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

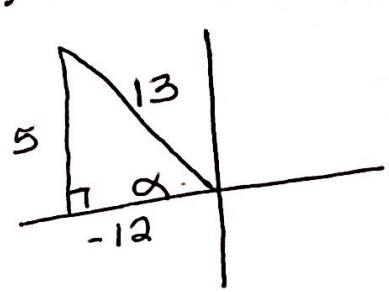
$1+\cot\alpha\tan\beta=1+\cot\alpha\tan\beta$

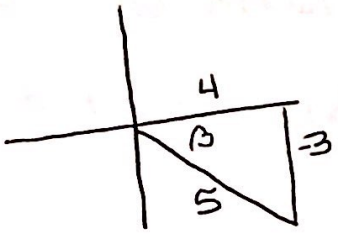
17.) $\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta}=1-\tan\alpha\tan\beta$

$\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$

$1-\tan\alpha\tan\beta=1-\tan\theta\tan\beta$

18) Draw Δ α QII β QIV





$\sin(\alpha-\beta)$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\frac{5}{13}\left(\frac{4}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right)$

$\frac{20}{65} - \frac{36}{65} = \frac{-16}{65}$

$\cos(\beta+\alpha)$

$\cos\beta\cos\alpha - \sin\beta\sin\alpha$

$= \frac{-33}{65}$

$\tan(\alpha-\beta)$

$= \frac{16}{63}$