Name: _____ Trig Identities

Prove the identities. *Remember only work on one side!

1.) $\cot^2 x + 1 = \csc^2 x$

$\frac{\cos^2 x}{\sin^2 x} + 1 = \downarrow$

$\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x = \csc^2 x$

2.) $\tan x + \cot x = \sec x \csc x$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \csc x$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \downarrow$

$= \frac{1}{\cos x \sin x} = \sec x \csc x = \sec x \csc x$

3.) $1 - 2\cos^2 x = 2\sin^2 x - 1$

$= 1 - 2(1 - \sin^2 x) = \downarrow$

$= 1 - 2 + 2\sin^2 x = \downarrow$

$= -1 + 2\sin^2 x = \downarrow$

$= 2\sin^2 x - 1 = \downarrow$

4.) $\sin x(\sec x - \csc x) = \tan x - 1$

$= \sin x \sec x - \sin x \csc x = \downarrow$

$= \sin x \cdot \frac{1}{\cos x} - \sin x \cdot \frac{1}{\sin x} = \downarrow$

$= \frac{\sin x}{\cos x} - 1 = \downarrow$

$= \tan x - 1 = \tan x - 1$

5.) $(1 + \sin\theta)^2 = 2(1 + \sin\theta) - \cos^2\theta$

$= (1 + 2\sin\theta + \sin^2\theta) = \downarrow$

$= 1 + 2\sin\theta + 1 - \cos^2\theta = \downarrow$

$= 2 + 2\sin\theta - \cos^2\theta = \downarrow$

$= 2(1 + \sin\theta) - \cos^2\theta =$

6.) $\frac{1-\tan\theta}{1+\tan\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}$

$= 1 - \frac{\frac{1}{\cot\theta}}{1 + \frac{1}{\cot\theta}} = \downarrow$

$= \frac{\cot\theta - 1}{\frac{\cot\theta}{\cot\theta}} = \downarrow$

$\frac{\cot\theta + 1}{\cot\theta}$

$\frac{\cot\theta - 1}{\cot\theta}, \frac{\cot\theta}{\cot\theta + 1} = \downarrow$

$= \frac{\cot\theta - 1}{\cot\theta + 1} = \downarrow$

7.) $\frac{\tan x + 1}{\cot x + 1} = \frac{\sec x}{\csc x}$

$= \frac{\frac{\sin x}{\cos x} + 1}{\frac{\cos x}{\sin x} + 1} = \downarrow$

$= \frac{\sin x + \cos x}{\cos x} \cdot \frac{\sin x}{\cos x + \sin x} = \downarrow$

$= \frac{\sin x}{\cos x} = \downarrow$

$= \frac{\sin x + \cos x}{\cos x} / \frac{\cos x + \sin x}{\sin x} = \downarrow$

$= \frac{1}{\csc x} / \frac{1}{\sec x} = \frac{\sec x}{\csc x} = \downarrow$

8.) $\tan x \sin x = \sec x - \cos x$

$= \frac{\sin x}{\cos x} \cdot \sin x = \downarrow$

$= \frac{\sin^2 x}{\cos x} = \downarrow$

$= \frac{1 - \cos^2 x}{\cos x} = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sec x - \cos x$

9.) $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$

$= \frac{1 - \sin x + 1 + \sin x}{(1 + \sin x)(1 - \sin x)} = \downarrow$

$= \frac{2}{1 - \sin^2 x} = \downarrow$

$= \frac{2}{\cos^2 x} = \downarrow$

$= 2\sec^2 x = \downarrow$

10.) $\frac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

$= \frac{\sec^2 x}{\tan^2 x} = \downarrow$

$= \frac{1/\cos^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$

11.) $\frac{\sec x}{1+\csc x} = \frac{\tan x}{1+\sin x}$

$= \frac{\frac{1}{\cos x}}{1 + \frac{1}{\sin x}} = \frac{1/\cos x}{\frac{\sin x + 1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{(\sin x + 1)}$

$= \frac{\sin x}{\cos x(\sin x + 1)} = \frac{\tan x}{\sin x + 1}$

12.) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$

$= \frac{(1 + \sin x)(1 + \sin x) + \cos x \cos x}{\cos x(1 + \sin x)}$

$= \frac{1 + 2\sin x + (\sin^2 x + \cos^2 x)}{\cos x(1 + \sin x)}$

$= \frac{2 + 2\sin x}{\cos x(1 + \sin x)} = \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} = 2\sec x$

13.) $\frac{\sin^2 x}{1-\cos x} = 1+\cos x$

$= \frac{1-\cos^2 x}{1-\cos x}$

$= \frac{(1+\cos x)(1-\cos x)}{1-\cos x} = 1+\cos x$

14.) $\frac{\sin x+\cos x}{\sec x+\csc x} = \frac{\cos x}{\csc x}$

$= \frac{\sin x+\cos x}{\frac{1}{\cos x}+\frac{1}{\sin x}}$

$= \frac{\sin x+\cos x}{\frac{\sin x+\cos x}{\sin x\cos x}}$

$= \frac{\sin x+\cos x}{1} \cdot \frac{\sin x\cos x}{\sin x+\cos x}$

$= \frac{1}{\csc x} \cdot \cos x = \frac{\cos x}{\csc x}$

15.) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$\sin^2 x - \cos^2 x = \downarrow$ (1)

16.) $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x$

$= \frac{1-\sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x} = \cos x = \frac{1}{\sec x}$

17.) $\frac{1+\cos x}{\sin x} = \csc x + \cot x$

$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$= \csc x + \cot x$

18.) $\frac{\sec\theta \sin\theta}{\tan\theta+\cot\theta} = \sin^2\theta$

$= \frac{\frac{1}{\cos\theta} \cdot \sin\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$

$= \frac{\sin\theta/\cos\theta}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta\cos\theta}{1}$

$= \sin^2\theta = \sin^2\theta$

19.) $\frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta} = 1$

$= \frac{1/\cos\theta}{\cos\theta} - \frac{\sin\theta/\cos\theta}{\cos\theta/\sin\theta}$

$= 1/\cos^2\theta - \sin^2\theta/\cos^2\theta$

$= \cos^2\theta/\cos^2\theta = 1 = 1$

20.) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

21.) $\csc^2 x \tan^2 x - 1 = \tan^2 x$

$= \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1$

$= 1/\cos^2 x - 1$

$= \frac{1-\cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

22.) $\frac{\sec^2\theta}{\sec^2\theta-1} = \csc^2\theta$

$\frac{\sec^2\theta}{\tan^2\theta} = \frac{1/\cos^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} =$

$\frac{1}{\sin^2\theta} = \csc^2\theta$

23.) $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$= (\sec^2 x - 1)\sin^2 x$

$= \sec^2 x \sin^2 x - \sin^2 x$

$= \frac{1}{\cos^2 x} \cdot \sin^2 x - \sin^2 x$

$= \tan^2 x - \sin^2 x$

24.) $\left(\frac{\sec x-1}{\sec x-1}\right) \frac{\sec x+1}{\tan x} = \frac{\tan x}{\sec x-1}$

$= \frac{\sec^2 x - 1}{(\sec x-1)(\tan x}$

$= \frac{\tan^2 x}{(\sec x-1)(\tan x)} = \frac{\tan x}{\sec x-1}$