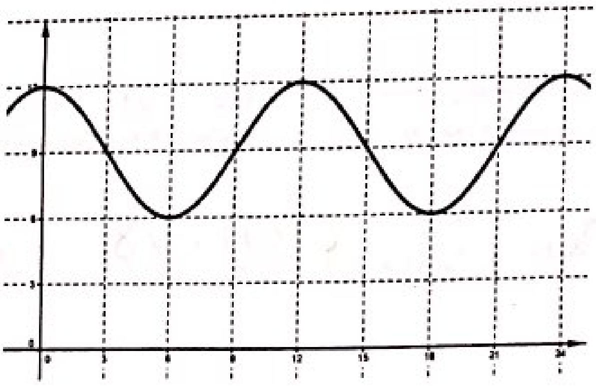
Name ____________________

Sinusoidal Applications

1. The initial behavior of the vibrations of the note E above middle C can be modeled by $y = 0.5 \sin 660\pi t$.
   a. What is the amplitude of this model? $1/2$
   b. What is the period of this mode? $1/330$
2. If the equilibrium point is $y = 0$, then $y = -4\cos(\frac{\pi}{6}t)$ models a buoy bobbing up and down in the water.
   a. What is the period of the function? 12
   b. What is the location of the buoy at $t = 10$. -2 ft
3. A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by $y = -3\cos(\frac{5\pi}{3}t) + 3.5$, where t is the time measured in seconds.
   a. What is the highest point reached by the knot? 6.5 ft
   b. What is the lowest point reached by the knot? $1/2$ ft
   c. What is the period of the model? $6/5$ sec
   d. According to the model, find the height of the knot after 25 seconds. 2 ft
4. The function $y = 25\sin(\frac{\pi}{6}t) + 60$, where t is in months and $t = 0$ corresponds to April 15, models the average high temperature in degrees Fahrenheit in Centerville.
   a. Find the period of the function. 12 Months
   b. What does the period **represent**? Temp repeats after 1 year
   c. What is the maximum high temperature? 85° F
   d. **When** does the maximum occur? t=3 so July 15
5. The figure shows the depth of water at the end of a boat dock. The depth is 6 feet at low tide and 12 feet at high tide. On a certain day, low tide occurs at 6 A.M. and high tide occurs at noon. If y represents the depth of the water x hours after midnight, use a **cosine function** of the form $y = A\cos Bx + D$ to model the water's depth.



$y = 3\cos(\frac{\pi}{6}t) + 9$

... average seated adult breathes in and out every 4 seconds. The average minimum ... cubic liters, and the average maximum amount of air in the lunges is 0.82 cubic liters. Suppose the lunges have a minimum amount of air at $t = 0$, where t is the time in seconds.

a. Write a function that models the amount of air in the lungs.

$y = -0.37\cos(\frac{\pi}{2}x) + .45$

b. Determine the amount of air in the lungs at 5.5 seconds.

0.712 liters^3

7. A Ferris wheel at an amusement park has riders get on the ride 3 m above the ground. The highest point of the ride is 23 meters above the ground. The ride takes 40 seconds for one complete revolution. Write the equation that models the height of the Ferris wheel over time.

$y = -10\cos(\frac{\pi}{20}t) + 13$

8. A Ferris wheel has a diameter of 80 feet. Riders enter the Ferris wheel at its lowest point, 6 feet above the ground at time t = 0 seconds. One complete rotation takes 67 seconds. Write a function modeling a riders height, h(t), at t seconds.

$y = -40\cos(\frac{2\pi}{67}t) + 46$

9. **Sam and Dan are being dared to ride the Ferris wheel. The height h (in feet) above the ground at any time t (in seconds) can be modeled by:** $y = 40\cos\left(\frac{\pi}{20}t + \frac{\pi}{2}\right) + 50$

a. Find the amplitude and period.

40

b. The Ferris wheel turns for 160 seconds before it stops to let Sam and Dan get off. How many times will they go around?

4 times

c. What are the minimum and maximum heights for Sam and Dan?

10ft 90ft

10. Suppose a Ferris wheel has a radius of 20 feet and operates at a speed of 3 revolutions per minute. The bottom car is 4 feet above the ground. Write a model for the height of a person above the ground whose height when t = 0 is the minimum.

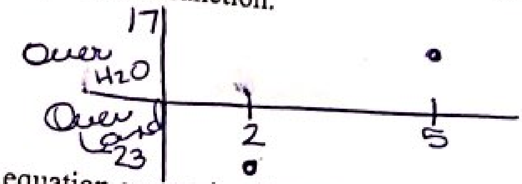
$y = -20\cos(\frac{\pi}{10}t) + 24$

11. A Ferris Wheel has a diameter of 50 feet and it takes 40 seconds to make a complete revolution. The bottom of the car is 3 feet above the ground. Write a model for the height of a person above the ground whose height is 8 feet off the ground when the ride begins.

$y = -25\cos(\frac{\pi}{20}(t-8)) + 28$

Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model his movement mathematically and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t and that y is positive when Tarzan is over water and negative when he is over land. Jane finds that when $t = 2$, Tarzan is at the end of his swing, where $y = -23$. She finds that when $t = 5$, he reaches the other end of his swing and $y = 17$.

a. Sketch a graph of this function.



b. Write an equation expression Tarzan's distance from the river bank in terms of t.

$y = -20\cos\left(\frac{\pi}{3}(x-2)\right) - 3$

c. Find y when t = 2.8 seconds

-16.38 ft

d. Where was Tarzan when Jane started her stopwatch?

7

13. Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time t = 0 years. A minimum number of 200 foxes appeared with t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

a. Sketch a graph of this sinusoid.

b. Write an equation expressing the number of foxes as a function of time.

$y = -300\cos\left(\frac{5\pi}{11}(x-2.9)\right) + 500$

c. Predict the fox population when t = 7 years.

≈ 227 foxes

d. Foxes are declared an endangered species when their population drops below 300. Between what two positive values of t where the foxes first endangered?

2.31 & 3.49 years

14. Sarah is sitting in an inner-tube in a wave pool. Her height above the bottom of the pool varies sinusoidally with time. At time t = 2 seconds Sarah is at the top of a wave, 3.5 m from the bottom of the pool. At t = 4 seconds she is at the bottom of the wave, 2.5 m from the bottom of the pool.

a. Sketch a graph of this sinusoid.

b. Write an equation to model Sarah's height.

$y = \frac{1}{2}\cos\left(\frac{\pi}{2}(t-2)\right) + 3$

c. Determine Sarah's height after 10 seconds.

3.5 ft