Bellwork

Solve.

1) logx + log(x - 3) = 1

$x = 5 \quad \cancel{x = -2}$

$\log_{10} x(x-3) = 1$

$10^1 = x^2 - 3x$

2) logx - log(x + 2) = log4

$\cancel{x = -8/3} \quad \phi$

$(x-5)(x+2) = 0$

$x^2 - 3x - 10 = 0$

3) $17^{x-7} + 6 = 40$

$\log_{17}(34) + 7 = x$

$x = 8.245$

4) $4^{3x-2} = 12^{5x}$

$x = -0.335$

$x = \frac{2\log 4}{3\log 4 - 5\log 12}$

$\log 4^{(3x-2)} = \log 12^{(5x)}$

Homework Questions

$(3x-2)\log 4 = 5x\log 12$

$3x\log 4 - 2\log 4 = 5x\log 12$

$-2\log 4 = 5x\log 12 - 3x\log 4$

$-2\log 4 = x(5\log 12 - 3\log 4)$

$\frac{-2\log 4}{5\log 12 - 3\log 4} = x$

Exponential Formulas

Growth: $y = P(1+r)^t$

Decay: $y = P(1-r)^t$

Compound Interest: $y = P\left(1+\frac{r}{n}\right)^{nt}$

Compounded Continuously: $y = Pe^{rt}$

Half-Life: $y = P\left(\frac{1}{2}\right)^{t/h}$

$\boxed{\text{Final Amount}}$

$\boxed{\text{Initial Amount}}$

$\boxed{\text{Eulers} \sim 2.718}$

$\boxed{\text{half Life}}$

Percentage $\boxed{\text{rate}}$ (decimal)

$\boxed{\text{time}}$

$\boxed{\text{number of times compounded per year}}$

Annually $n=1$

SemiAnnually $n=2$

Quarterly $n=4$

Monthly $n=12$

Weekly $n=52$

Daily $n=365$

A small town has a population of 8,702 (P) in the year 2000 and is growing at a rate of 2.8% per year. ($r=.028$) What is the expected population in the year 2030? $t=30$

$$y = 8702(1+.028)^{30}$$

$$= 19925.65 = 19,926 \text{ people}$$

How long will it take for the population to reach 15,000?
final

$$15,000 = 8702(1+.028)^t$$

$$\frac{15000}{8702} = (1.028)^t$$

$$\log_{1.028}\left(\frac{15000}{8702}\right) = t$$

$t = 19.7$ years

A population of 1200 deer is dying at a rate of 7% per year. How many deer are expected after 10 years? $y=P(1-r)^t$

$$=1200(1-.07)^{10}$$

$$=580.7 \text{ deer} \approx 581$$

How many years until 100 deer remain?

$$100=1200(.93)^t$$

$$\frac{100}{1200}=(.93)^t$$

$$\log_{.93}\left(\frac{100}{1200}\right)=t \qquad 34.2 \text{ years}$$

Suppose I invest \$300 into an account that earns 2% interest compounded every 6 months. How much money will I have after 5 years? $n=2$

$$y=P\left(1+\frac{r}{n}\right)^{nt} \qquad y=300\left(1+\frac{.02}{2}\right)^{2(5)}$$

$$\$331.39$$

Suppose Kaylee invests \$500 into an account that earns 9% interest compounded monthly. $n=12$

How long will it take her to earn \$1000?

$$1500=500\left(1+\frac{.09}{12}\right)^{12t}$$

$$3=\left(1+\frac{.09}{12}\right)^{12t}$$

$$\log_{\left(1+\frac{.09}{12}\right)}(3)=12t$$

$$t \approx 12.3 \text{ years}$$

You invest some money into an account that earns 4% interest compounded continuously. How long will it take for you to triple your money?

$y = Pe^{rt}$

$300 = 100e^{rt}$

$3 = e^{.04t}$

$\ln(3) = .04t$

$t \approx 27.5$ years

If James invests ~~\$400~~ into an account that compounds interest quarterly, at what rate does he need the interest to be if he wants to double his investment in 3 years?

$2 = (1 + \frac{r}{4})^{4(3)}$

$\sqrt[12]{2} = 1 + r/4$

$(\sqrt[12]{2} - 1) = r/4$

$4(\sqrt[12]{2} - 1) = r$

$r = .238$

Selenium-83 has a half-life of 25 minutes. How many minutes would it take for a 10 mg sample to decay and have only 1.20 mg of it remain?

P

y

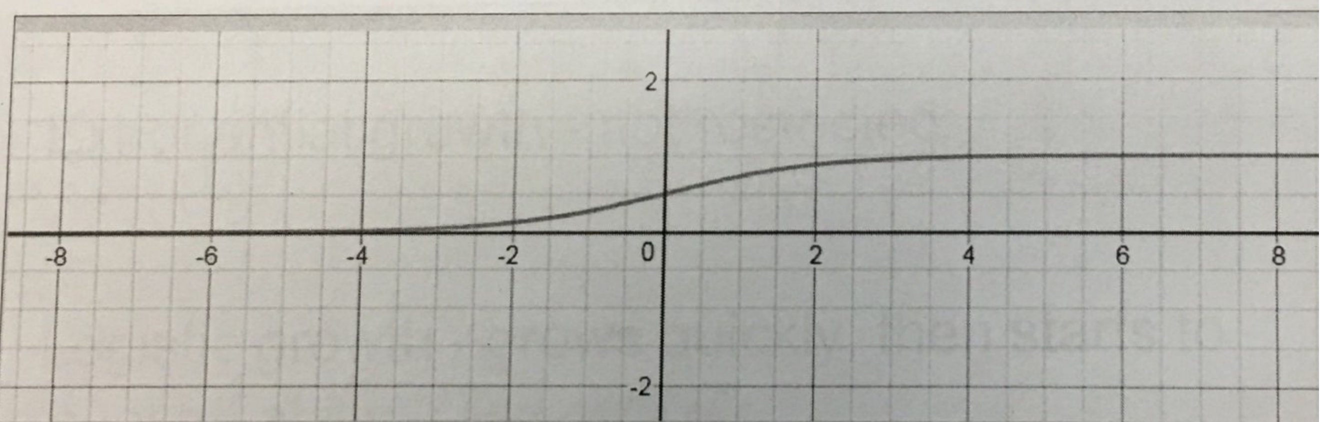
$1.20 = 10(\frac{1}{2})^{t/25}$

$\frac{1.2}{10} = (\frac{1}{2})^{t/25}$

$\log_{1/2}(\frac{1.2}{10}) = t/25$

≈ 76.5 minutes

Logistic Functions



$f(x) = \dfrac{1}{1 + e^{-x}}$

Domain: $(-\infty, \infty)$

Range: $(0, 1)$

Asymptotes: $y = 0$ $y = 1$

Increasing: $(-\infty, \infty)$

Decreasing: never

Logistic Growth - models restricted populations

$$f(x) = \frac{c}{1 + ab^x} \qquad \text{OR} \qquad f(x) = \frac{c}{1 + ae^{-kx}}$$

where a, b, c, and k are positive constants, b<1 & c is called the limit to growth.

Exponential growth - not restricted

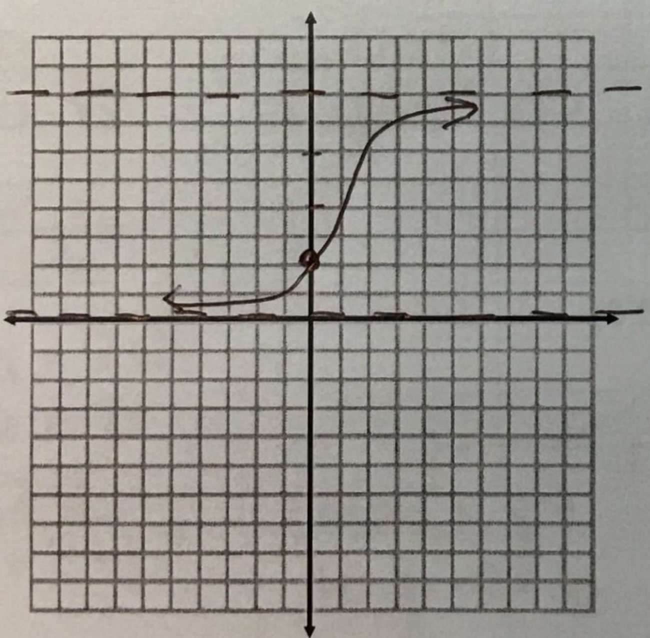
Logistic growth - grows quickly, then starts to level off at it's asymptote.

*All logistic growth functions have graphs like the basic logistic function where the end behavior is at the HAs.

*All logistic growth functions are bounded by asymptotes y = <u>O</u> and y = <u>C</u>

*All logistic growth functions have a range of <u>(0, C)</u>

Sketch the following logistic growth functions, identify the y-intercept and asymptotes.



$$f(x) = \frac{8}{1+3*.7^x}$$

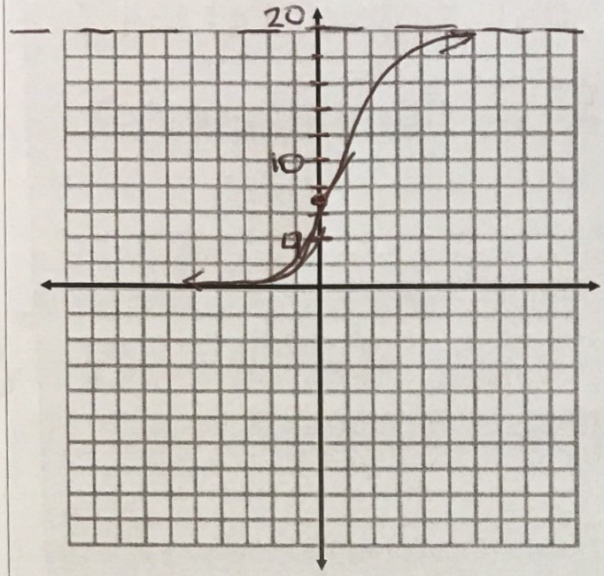
$y=0 \quad y=8$

Initial value (y-in

$x=0$

$$y=\frac{8}{1+3\cdot 7^0}=2$$

Sketch the following logistic growth functions, identify the y-intercept and asymptotes.



$$f(x) = \frac{20}{1+2e^{-3x}}$$

$y=0$

$y=20$

y-int

$$y = \frac{20}{1+2} = \frac{20}{3} \sim 6.67$$

Write an equation for the logistic graph.

1st

$$y = \frac{c}{1+a\cdot b^x}$$

$$y = \frac{20}{1+a\cdot b^x}$$ use (0,5) to find a

$$5 = \frac{20}{1+a\cdot b^0}$$

$5+5a=20$

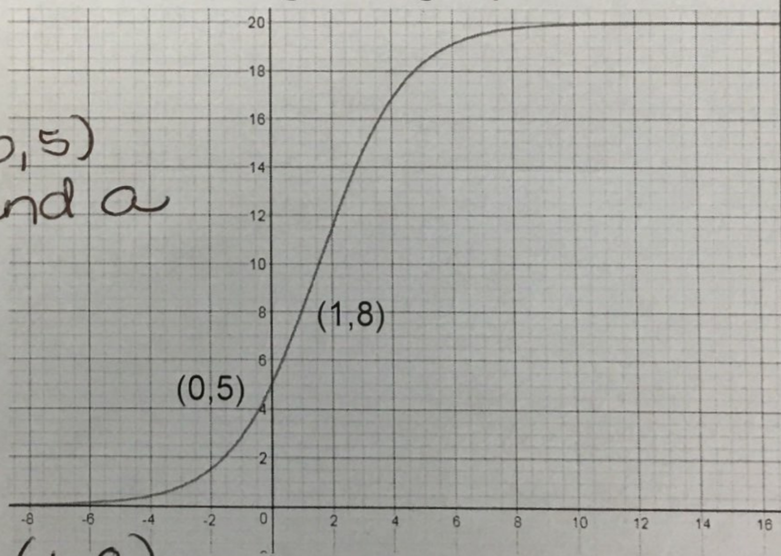
$a=3$

$$y = \frac{20}{1+3\cdot b^x}$$ use (1,8) to find b

$$8 = \frac{20}{1+3\cdot b^1}$$

$8+24b=20$

$b=1/2$



$$y = \frac{20}{1+3\cdot(1/2)^x}$$

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Find the logistic function given the following.

Initial value = 16

Limit to growth = 128

Passes through (5, 32)

$y = \frac{128}{1+a\cdot b^x}$ use (0,16)

$16 = \frac{128}{1+a b^0}$

$16+16a=128$

$a=7$

use (5,32)

$32 = \frac{128}{1+7\cdot b^5}$

$b = \sqrt[5]{3/7}$

$y = \frac{128}{1+7\cdot\left(\sqrt[5]{3/7}\right)^x}$

Solve the following logistic equation.

$$\frac{50}{1+10e^{-3x}} = 40$$

$50 = 40(1+10e^{-3x})$

$\frac{5}{4} = 1+10e^{-3x}$

$\frac{1}{4} = 10e^{-3x}$

$\frac{1}{40} = e^{-3x}$

$\ln(1/40) = -3x$

$-3,689 = -3x$

$x = 1.2296$

Solve the following logistic equation.

$$\frac{30}{1+5e^{-2x}}=10$$

$\frac{30}{10} = 1+5e^{-2x}$

$2 = 5e^{-2x}$

$\frac{2}{5} = e^{-2x}$

$\ln(2/5) = -2x$

$-.916 = -2x$

$x = .4581$

The function

$$f(t) = \frac{30{,}000}{1 + 20e^{-1.5t}}$$

describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.

a. How many people became ill with the flu when the epidemic began?
b. How many people were ill by the end of the fourth week?
c. What is the limiting size of $f(t)$, the population that becomes ill?

a) $t=0$ $\frac{30{,}000}{21} \approx$ 1428 ppl

b) $t=4$ 28,522 ppl

c) 30,000 ppl

In a learning theory project, psychologists discovered that $f(t) = \dfrac{0.8}{1 + e^{-0.2t}}$ is a model for describing the proportion of correct responses after t trials.

a. Find the proportion of correct responses prior to learning trials taking place.

$\frac{.8}{2} \approx 0.4$

b. Find the proportion of correct responses after 10 learning trials.

c. What is the limiting size of f(t)?

A high school has 1200 students. Kim, Emily, and Hunter start a rumor about Mrs. Barnhart, which spreads logistically so that $S(t) = \dfrac{1200}{1 + 39e^{-.9t}}$ models the number of students who have heard the rumor by the end of t days.

a. How many students have heard the rumor by the end of day 0?

$\frac{1200}{40} = 30$

b. How long does it take for 1000 students to hear the rumor?

$1000 = \frac{1200}{1+39e^{-.9t}}$

$t \approx 5.86$ days