A **series** is the expression for the sum of the terms of a sequence.

Finite sequences and series have terms that you can count individually from 1 to a final whole number *n*.

Infinite sequences and series continue without end. You can indicate an infinite sequence with an ellipsis.

Finite vs. Infinite

Finite sequence	Finite Series
6, 9, 12, 15, 18	6 + 9 + 12 + 15 + 18

Infinite sequence	Infinite Series
3, 7, 11, 15,...	3 + 7 + 11 + 15 + ...

Example 1:

Find the sum of the numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9

One way to find the sum is to add each term individually:

1+2+3+4+5+6+7+8+9 = 45

Is this always feasible? no!

The sum of a ***finite*** arithmetic series:

#terms 1st term

$$S_n = \frac{n}{2}(a_1 + a_n)$$

sum of n terms

Last term

Find the sum of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 using this formula.

$S_n = \frac{9}{2}(1+9)$

$= 45$

1. Find the sum of the first 10 numbers of the series $23 + 25 + 27 + 29 + \ldots + 41$.

$S_{10} = \frac{10}{2}(23+41) = 320$

2. Find the sum of: $-2 + 2.5 + 7 + 11.5 + \ldots + 268$

$a_n = a_1 + d(n-1)$

$268 = -2 + 4.5(n-1)$

$n = 61$

$S_n = \frac{61}{2}(-2+268)$

$= 8113$

3. $a_1 = 12$ $n = 8$ $a_n = -23$

$S_8 = \frac{8}{2}(12-23) = -44$

Determine the number of terms for the given series.

$-2 + 2.5 + 7 + \ldots,\ S_n = 5194$

$5194 = \frac{n}{2}(-2 + a_n)$

$a_n = -2 + 4.5(n$

$= 4.5n - 6.$

$5194 = \frac{n}{2}(-2 + 4.5n - 6.5)$

$10388 = n(4.5n - 8.5)$

$0 = 4.5n^2 - 8.5n - 10388 -$

Quad for $n = 49$

* n has to be positive

1. Given $S_n = 822$, $n = 12$, and $a_1 = 8$, find a_n.

$822 = \frac{12}{2}(8 + a_n)$

$a_n = 129$

2. Given $a_1 = 12$ and $a_n = 86$, and $S_n = 931$, find n.

$931 = \frac{n}{2}(12+86)$

$n = 19$

Summation Notation

Last term # position

$$\sum_{n=i}^{j} a_n$$

Explicit Eq

1st term #

*Also called Sigma notation

Evaluate $\sum_{n=1}^{85} (n-1)$

$S = \frac{85}{2}(0+84)$

$= 3570$

Evaluate $\sum_{n=12}^{32} (25-2n)$

$32-12+1$

$S = \frac{21}{2}(1-39) = -399$

not Arithmetic
find by hand

Evaluate the following.

1. $\sum_{m=1}^{4} (m^2 - 2)$

$-1, 2, 7, 14$
$= 22$

2. $\sum_{n=2}^{6} 3n^2$

$12, 27, 48, 75, 108$
$= 270$

3. $\sum_{n=0}^{6} (4n^2 + 1)$

$1, 9, 17, 37, 65, 101, 145$
$= 371$

Write the following series using sigma notation.

1. 5 + 9 + 13 ... + 33 ← 1st what posit.

$\sum_{n=1}^{8} (4n+1)$

$33 = 5 + 4(n-1)$
$n = 8$

2. 7, 1, -5, -11, ... , -53

$\sum_{1}^{11} (-6n+13)$

$-53 = 7 + -6(n-1)$
$\frac{60}{-6} =$ $n = 11$
$a_n = 7 - 6(n-1)$
$a_n = 13 - 6n$

A geometric series is the expression for the sum of a geometric sequence.

As with arithmetic series, we can use a formula to evaluate geometric finite series.

1st term, ratio, # term

$$S_n = \frac{a(1 - r^n)}{1 - r} \qquad r \neq 1$$

Sum of n terms

Use the formula to evaluate the series

3 + 6 + 12 + 24 + 48 + 96.

$S = \frac{3(1-2^6)}{1-2} = 189$

Find the sum of the following series.

1. $a_1 = 60$ $\quad n = 6$ $\quad r = 1/2$

$S = \frac{60(1-(\frac{1}{2})^6)}{1-\frac{1}{2}} = 118.125$

2. Find the sum of: 2, 4, 8, ... to 8 terms

$S = \frac{2(1-2^8)}{1-2} = 510$

Find the sum of $\sum_{n=1}^{8} 4 \bullet 3^{n-2}$

plug in 1 to get 1st term

1st term $= 4/3$

$S = \frac{4/3(1-3^8)}{1-3}$

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