

Even and Odd Functions

Terminology	Definition	Illustration	Type of symmetry of graph
f is an even function	$f(x) = f(-x)$ for every x in the domain	$y = f(x) = x^2$	With respect to the y-axis
f is an odd function	$-f(x) = f(-x)$ for every x in the domain	$y = f(x) = x^3$	With respect to the origin

Determine whether f is even, odd or neither even nor odd.

1. $f(x) = 5x^3 + 2x$

$$f(-x) = -5x^3 - 2x$$

$$-f(x) = -5x^3 - 2x$$

since $f(-x) = -f(x)$

\therefore odd

2. $f(x) = |x| - 3$

$$f(-x) = |-x| - 3 = |x| - 3$$

$$f(-x) = f(x)$$

\therefore even

3. $f(x) = 3x^4 + 2x^2 - 5$

$$f(-x) = 3x^4 + 2x^2 - 5$$

$$f(-x) = f(x)$$

even

4. $f(x) = 7x^5 - 4x^3$

$$f(-x) = -7x^5 + 4x^3$$

$$-f(x) = -7x^5 + 4x^3$$

$$f(-x) = -f(x)$$

odd

5. $f(x) = 8x^3 - 3x^2$

$$f(-x) = -8x^3 - 3x^2$$

$$-f(x) = -8x^3 + 3x^2$$

$$f(-x) \neq f(x) \neq -f(x)$$

Neither

6. $f(x) = 12$

$$f(-x) = 12$$

$$f(-x) = f(x)$$

even

7. $f(x) = \frac{1}{x}$

$$f(-x) = -\frac{1}{x}$$

$$-f(x) = -\frac{1}{x}$$

$$f(-x) = -f(x)$$

odd

8. $f(x) = 3x^2 - 5x + 1$

$$f(-x) = 3x^2 + 5x + 1$$

$$-f(x) = -3x^2 + 5x - 1$$

Neither

$$f(-x) \neq f(x) \neq -f(x)$$

9. $f(x) = \sqrt{x^2 + 4}$

$$f(-x) = \sqrt{x^2 + 4}$$

$$f(-x) = f(x)$$

even

10. $f(x) = \sqrt[3]{x^3 - x}$

$$f(-x) = -\sqrt[3]{x^3 - x}$$

$$-f(x) = -\sqrt[3]{x^3 - x}$$

odd

$$f(-x) = -f(x)$$

	Function	Even	Odd	Neither
1	$f(x) = 5x^2 - 4x + 3$			✓
2	$g(x) = 3x^3 - 4x$		✓	
3	$h(x) = -2x^4 + 3x^2$	✓		
4	$f(x) = -2x^5 + 3x^3 - 5x^2$			✓
5	$n(x) = \sqrt{x^2 + 9}$	✓		
6	$s(x) = 3x^{\frac{1}{3}}$		✓	
7	$g(x) = \sqrt{1 - x^2}$	✓		
8	$h(x) = x^6 + 2x^4 - 3x^2 - 4$	✓		
9	$m(x) = (x - 2)^2$			✓
10	$p(x) = 2x\sqrt{x^2 + 4}$		✓	

Expression Check: $4(\# \text{ of Evens}) + 3(\# \text{ of Odds}) - (\# \text{ of Neither}) = 22$

$$4() + 3() - () =$$