

Final Exam Review

Describe the transformations necessary to transform the graph of $f(x)$ into that of $g(x)$.

1) $f(x) = x^2$
 $g(x) = -(x+2)^2$

reflect x
 L2

2) $f(x) = \sqrt{x}$
 $g(x) = \sqrt{\frac{1}{3}x} - 2$

H. stretch by 3
 ↓ 2

3) $f(x) = |x|$
 $g(x) = \frac{1}{2} \cdot |x-1|$

V comp by 1/2
 R1

4) $f(x) = \sqrt{x}$
 $g(x) = \sqrt{-(x-3)} + 2$

H2 R3
 reflect y

Convert each pair of polar coordinates to rectangular coordinates.

5) $(3, 315^\circ)$ $(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2})$

6) $(2, \frac{\pi}{3})$ $(1, \sqrt{3})$

Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \leq \theta < 2\pi$.

7) $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ $(1, \frac{4\pi}{3})$

8) $(\sqrt{2}, -\sqrt{2})$ $(2, \frac{7\pi}{4})$

Convert each equation from rectangular to polar form.

9) $(x+3)^2 + (y-1)^2 = 10$

$r = -6\cos\theta + 2\sin\theta$

10) $x^2 + (y-2)^2 = 4$

$r = 4\sin\theta$

11) $(x-2)^2 + (y+1)^2 = 5$

$r = 4\cos\theta - 2\sin\theta$

12) $(x+2)^2 + (y-1)^2 = 5$

$r = -4\cos\theta + 2\sin\theta$

Convert each equation from polar to rectangular form. Identify the conic section.

13) $r = \frac{2}{3 + 2\cos\theta}$

Ellipse
 $5x^2 + 9y^2 + 8x - 4 = 0$

14) $r = -2\sin\theta$

Circle
 $x^2 + (y+1)^2 = 1$

15) $r = \frac{6}{2 - \sin \theta}$

Ellipse

$4x^2 + 3y^2 - 12y - 36 = 0$

16) $r = 2\cos \theta + 6\sin \theta$ Circle

$(x-1)^2 + (y-3)^2 = 10$

Write each pair of parametric equations in rectangular form.

17) $x = 5\cos t, y = 2\sin t$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

18) $x = 5\cos t + 1, y = 4\sin t - 1$

$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{16} = 1$

19) $x = -2t - 2, y = t^2 - 4$

$y = \frac{x^2}{4} + x - 3$

20) $x = 2t - 3, y = \frac{2t^2}{3} - 2t + \frac{3}{2}$

$y = \frac{x^2}{6}$

Identify the conic then find all applicable information (center, vertices, foci, asymptotes, covertices, directrix, vertex, etc).

21) $x^2 + y^2 - 4y - 60 = 0$

Circle

C(0, 2)
r=8

22) $9x^2 + 16y^2 + 72x - 320y + 448 = 0$

Ellipse

C(-4, 10)
V(8, 10), (-16, 10)
CoV(-4, 19), (-4, 1)
F(-4 + 3√7, 10), (-4 - 3√7, 10)

23) $x^2 - y^2 + 8x - 18y - 114 = 0$

hyperbola

C(-4, -9)
V(3, -9), (-11, -9)
F(-4 ± 7√2, -9)

Asy: $y = x - 5$
 $y = x - 13$

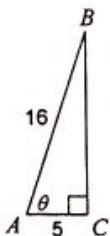
24) $-3x^2 - 60x + 7y - 307 = 0$

Parabola

V(-10, 1)
F(-10, 19/12)
Directrix $y = 5/12$

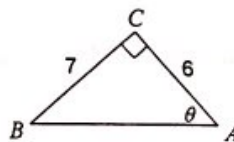
Find the measure of each angle indicated. Round to the nearest tenth.

25)



71.8°

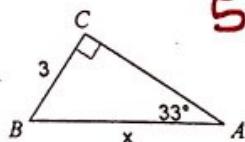
26)



49.4°

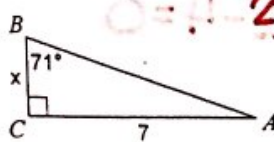
Find the measure of each side indicated. Round to the nearest tenth.

27)



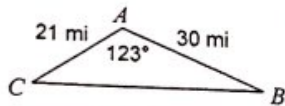
5.5

28)



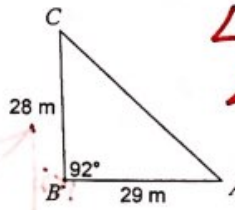
Solve each triangle. Round your answers to the nearest tenth.

29)



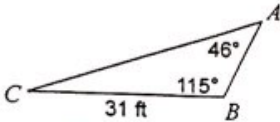
$\angle C = 34^\circ$
 $\angle B = 23^\circ$
 $a = 45 \text{ mi}$

30)



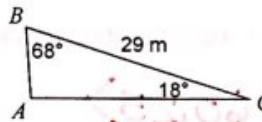
$\angle C = 45^\circ$
 $\angle A = 43^\circ$
 $b = 41 \text{ m}$

31)



$\angle C = 19^\circ$
 $b = 39.1 \text{ ft}$

32)



$\angle A = 94^\circ$
 $b = 27 \text{ m}$

Solve each equation for $0 \leq \theta < 2\pi$.

33) $2\sin \theta = \sqrt{3}$ $\pi/3$ $2\pi/3$

34) $3\cot \theta = -3$ $3\pi/4$ $7\pi/4$

35) $4\cos\left(\theta + \frac{3\pi}{4}\right) = 2$

$\frac{11\pi}{12}$; $\frac{19\pi}{12}$

36) $\sqrt{2} = 2\sin\left(\theta + \frac{3\pi}{2}\right)$

$\frac{3\pi}{4}$, $\frac{5\pi}{4}$

37) $\tan \theta + \sqrt{2}\tan \theta \sin \theta = 0$

$0, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$

38) $-2\cos^2 \theta + \cos \theta + 2 = 1$

$0, \frac{2\pi}{3}, \frac{4\pi}{3}$

39) $3\csc \theta = -2\sqrt{3}\csc \theta \cos \theta$

$\frac{5\pi}{6}, \frac{7\pi}{6}$

40) $4 = -\tan^2 \theta + 7$

$\pi/3, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Find the exact value of each expression.

41) $\cos^{-1} \left(-\frac{\sqrt{2}}{2}\right)$

$\frac{3\pi}{4}$

42) $\csc^{-1}(-\sqrt{2})$

$-\frac{\pi}{4}$

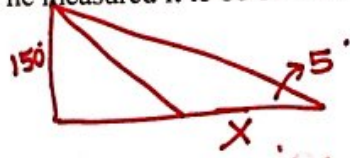
43) $\sec^{-1}\left(\tan \frac{\pi}{4}\right)$

$\pi/2$

44) $\tan^{-1}\left(\csc -\frac{\pi}{2}\right)$

$-\pi/4$

45) From a cliff 150 feet above a lake, Julio saw a boat sailing directly toward him. The angle of depression was 5° . A few minutes later, he measured it to be 11° . Find the distance the boat sailed between the two observations.



942.82 ft apart

Find the component form of the resultant vector. Then find the magnitude.

46) $f = \langle -12, -5 \rangle$
Find: $-5f$

$\langle 60, 25 \rangle$

47) Given: $T = \langle -5, -1 \rangle$ $X = \langle -3, -10 \rangle$
Find: $\sqrt{2} \cdot \overline{TX}$

$\langle 2\sqrt{2}, -9\sqrt{2} \rangle$

48) Given: $T = \langle 1, -6 \rangle$ $X = \langle -10, 9 \rangle$
 $Y = \langle 4, 2 \rangle$ $Z = \langle -10, 8 \rangle$
Find: $-\overline{TX} + \overline{YZ}$

$\langle -3, -9 \rangle$

49) $f = \langle -3, 10 \rangle$
 $g = \langle 3, -6 \rangle$
Find: $-f - g$

$\langle 0, -4 \rangle$

Find the dot product of the given vectors.

50) $u = \langle 3, -3 \rangle$
 $v = \langle 1, -8 \rangle$

27

51) $u = -8i + 5j$
 $v = -8i$

64

Find the component form of the resultant vector.

52) $a = \langle -8, 1 \rangle$
Unit vector in the direction of a

$\left\langle \frac{-8\sqrt{65}}{65}, \frac{\sqrt{65}}{65} \right\rangle$

53) $f = \langle -12, 16 \rangle$
Unit vector in the opposite direction of f

$\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

Write the recursive & explicit formula for each sequence.

54) $\frac{6}{5}, \frac{18}{25}, \frac{54}{125}, \frac{162}{625}, \frac{486}{3125}, \dots$

E: $a_n = \frac{6}{5} \left(\frac{3}{5}\right)^{n-1}$

R: $a_n = \frac{3}{5} a_{n-1}$

$a_1 = \frac{6}{5}$

55) $0, -2, -4, -6, -8, \dots$

E: $a_n = 2 - 2n$

R: $a_n = a_{n-1} - 2$

$a_1 = 0$

Evaluate each series.

$$56) \sum_{n=1}^7 (4n^2 - 4)$$

532

$$57) \sum_{a=2}^6 (a + 300)$$

1520

Determine the number of terms n in each series.

$$58) 18 + 23 + 28 + 33 \dots, S_n = 986 \text{ (Arithmetic)}$$

17

$$59) -2 - 12 - 72 \dots, S_n = -3110 \text{ (Geometric)}$$

5

Rewrite each series using sigma notation.

$$60) 4 + 16 + 64 + 256 + 1024$$

$$\sum_{n=1}^5 4^n \text{ or } \sum_{n=1}^5 4(4)^{n-1}$$

$$61) 10 + 15 + 20 + 25 + 30 + 35$$

$$\sum_{n=1}^6 (5n + 5)$$

Evaluate each limit.

$$62) \lim_{x \rightarrow 3^-} -\frac{3x}{x-3}$$

∞

$$63) \lim_{x \rightarrow -1^+} -\frac{x+1}{x^2-1}$$

$\frac{1}{2}$

$$64) \lim_{x \rightarrow 2} \frac{1}{x-2}$$

Dne

$$65) \lim_{x \rightarrow -4} \frac{x+4}{x^2+7x+12}$$

-1

$$66) \lim_{x \rightarrow 1^+} \frac{2x-2}{|x-1|}$$

2

$$67) \lim_{x \rightarrow -1^-} f(x), f(x) = \begin{cases} -x^2 - 6x - 9, & x \leq -1 \\ 0, & x > -1 \end{cases}$$

-4

$$68) \lim_{x \rightarrow 4} \frac{-x+4}{|-x+4|}$$

Dne

$$69) \lim_{x \rightarrow 2} f(x), f(x) = \begin{cases} \frac{x}{2} + \frac{3}{2}, & x < 2 \\ -x + 6, & x \geq 2 \end{cases}$$

Dne