Name: ____________________________

Analytic Geometry/Trigonometry Identities

Prove the identities.

1. $\cot^2 x + 1 = \csc^2 x$

$$\frac{\cos^2 x}{\text{sen}^2 x} + 1$$

$$\frac{\cos^2 x + \text{sen}^2 x}{\text{sen}^2 x}$$

$$\frac{1}{\text{sen}^2 x} = \csc^2 x = \csc^2 x$$

2. $\tan x + \cot x = \sec x \csc x$

$$\frac{\text{sen}\, x}{\cos x} + \frac{\cos x}{\text{sen}\, x}$$

$$\frac{\text{sen}^2 x + \cos^2 x}{\cos x\, \text{sen}\, x}$$

$$\frac{1}{\cos x\, \text{sen}\, x}$$

$$\sec x \csc x =$$

3. $1 - 2\cos^2 x = 2\sin^2 x - 1$

$$1 - 2(1 - \text{sen}^2 x)$$

$$1 - 2 + 2\,\text{sen}^2 x$$

$$-1 + 2\,\text{sen}^2 x$$

$$2\,\text{sen}^2 x - 1 =$$

4. $\sin x(\sec x - \csc x) = \tan x - 1$

$$\text{sen}\, x \sec x - \text{sen}\, x \csc x = \Downarrow$$

$$\text{sen}\, x\left(\frac{1}{\cos x}\right) - \text{sen}\, x\left(\frac{1}{\text{sen}\, x}\right)$$

$$\frac{\text{sen}\, x}{\cos x} - 1$$

$$\tan x - 1 =$$

$$\frac{1 - \tan x}{1 + \tan x}$$

5. $(1 + \sin x)^2 = 2(1 + \sin x) - \cos^2 x$

$$1 + 2\,\text{sen}\, x + \text{sen}^2 x =$$

$$1 + 2\,\text{sen}\, x + 1 - \cos^2 x =$$

$$2 + 2\,\text{sen}\, x - \cos^2 x =$$

$$2(1 + \text{sen}\, x) - \cos^2 x =$$

6. $\frac{1-\tan x}{1+\tan x} = \frac{\cot x - 1}{\cot x + 1}$

$$\frac{1 - \frac{\text{sen}\, x}{\cos x}}{1 + \frac{\text{sen}\, x}{\cos x}}$$

$$\frac{\frac{\cos x - \text{sen}\, x}{\cos x}}{\frac{\cos x + \text{sen}\, x}{\cos x}}$$

$$\frac{\cos x - \text{sen}\, x}{\cos x + \text{sen}\, x}$$

$$\frac{1 - \tan}{1 + \tan x}\left(\frac{1 - \frac{1}{\cot x}}{1 + \frac{1}{\cot x}}\right.$$

$$\frac{\frac{\cot x - 1}{\cot x}}{\frac{\cot x + 1}{\cot x}}$$

$$\frac{\cot x - 1}{\cot x + 1}$$

7. $\frac{\tan x+1}{\cot x+1} = \frac{\sec x}{\csc x}$

$$\frac{\frac{\sin x}{\cos x}+1}{\frac{\cos x}{\sin x}+1}$$

$$\frac{\frac{\sin x+\cos x}{\cos x}}{\frac{\cos x+\sin x}{\sin x}}$$

$$\frac{\cancel{\sin x+\cos x}}{\cos x}\cdot\frac{\sin x}{\cancel{\sin x+\cos x}}$$

$$\frac{\sin x}{\cos x}$$

$$\frac{\frac{1}{\csc x}}{\frac{1}{\sec x}}$$

$$= \frac{\sec x}{\csc x}$$

8. $\tan x \sin x = \sec x - \cos x$

$$\frac{\sin x}{\cos x}\cdot\sin x$$

$$\frac{\sin^2 x}{\cos x} = \frac{1-\cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$\sec x - \cos x$$

9. $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$

$$\frac{1-\sin x+1+\sin x}{1-\sin^2 x}$$

$$\frac{2}{\cos^2 x} = 2\sec^2 x$$

10. $\frac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

$$\frac{\sec^2 x}{\tan^2 x} = \frac{1/\cos^2 x}{\frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{1}{\cancel{\cos^2 x}}\cdot\frac{\cancel{\cos^2 x}}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$$

11. $\frac{\sec x}{1+\csc x} = \frac{\tan x}{1+\sin x}$

$$\frac{\frac{1}{\cos x}}{1+\frac{1}{\sin x}} = \frac{\frac{1}{\cos x}}{\frac{\sin x+1}{\sin x}} = \frac{1}{\cos x}\cdot\frac{\sin x}{(\sin x+1)}$$

$$\frac{\tan x}{\sin x+1}$$

12. $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$

$$\frac{1+2\sin x+\cancel{\sin^2 x}+\cancel{\cos^2 x}}{\cos x(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)} = \frac{2(\cancel{1+\sin x})}{\cos x(\cancel{1+\sin x})}$$

$$2\sec x$$

13. $\frac{\sin^2 x}{1-\cos x} = 1+\cos x$

$$\frac{1-\cos^2 x}{1-\cos x} = \frac{(1+\cos x)(\cancel{1-\cos x})}{\cancel{1-\cos x}}$$

$$1+\cos x = 1+\cos x$$

14. $\frac{\sin x+\cos x}{\sec x+\csc x} = \frac{\cos x}{\csc x}$

$$\frac{\sin x+\cos x}{\frac{1}{\cos x}+\frac{1}{\sin x}} = \frac{\sin x+\cos x}{\frac{\cos x+\sin x}{\cos x\sin x}}$$

$$\cancel{\sin x+\cos x}\left(\frac{\cos x\sin x}{\cancel{\cos x+\sin x}}\right)$$

$$\cos x\sin x = \cos x\left(\frac{1}{\csc x}\right)$$

$$= \frac{\cos x}{\csc x}$$

15. $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

$$(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)$$

$$1(\sin^2 x-\cos^2 x)$$