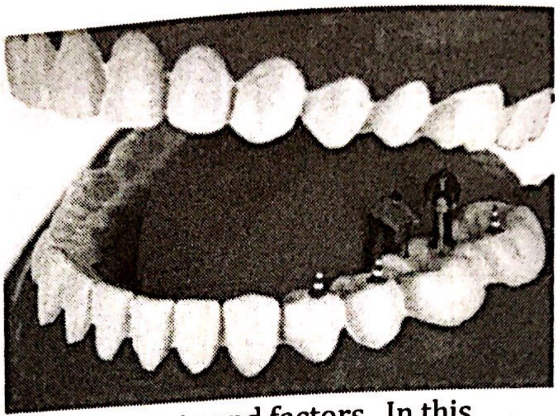
conjugate EQ - ha... can we ... all roots poly

Lesson 5: Getting to the Root of the Problem

A Solidify Understanding Task



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In *Building Strong Roots,* we learned to predict the number of roots of a polynomial using the Fundamental Theorem of Algebra and the relationship between roots and factors. In this task, we will be working on how to find all the roots of a polynomial given in standard form.

Let's start by thinking again about numbers and factors.

gether

1. If you know that 7 is a factor of 147, what would you do to find the prime factorization of 147? Explain your answer and show your process here:

number unfactored form

147
7 · 21
7 · 3

$3 \cdot 7^2$

2. How is your answer like a polynomial written in the form: $P(x) = (x-7)^2(x-3)$?

poly unfactored form

The process for finding factors of polynomials is exactly like the process for finding factors of numbers. We start by dividing by a factor we know and keep dividing until we have all the factors. When we get the polynomial broken down to a quadratic, sometimes we can factor it by inspection, and sometimes we can use our other quadratic tools like the quadratic formula.

Let's try it! For each of the following functions, you have been given one factor. Use that factor to find the remaining factors, the roots of the function, and write the function in factored form.

3. Function: $f(x) = x^3 + 3x^2 - 4x - 12$ Factor: $(x+3)$ Roots of function:

$x^2 - 4 = 0$ $x+3=0$

$x = -3$ | 1 3 -4 -12
-3 0 12
1 0 -4 0

$x = -3$
$x = 2$
$x = -2$

Factored form: $f(x) = (x+3)(x+2)(x-2)$

4. Function: $f(x) = x^3 + 6x^2 + 11x + 6$ Factor: $(x+1)$ Roots of function:

$x = -1$
$x = -2$
$x = -3$

Factored form: $f(x) = (x+3)(x+2)(x+1)$

5. Function: $f(x) = x^3 - 5x^2 - 3x + 15$ Factor: $(x-5)$ Roots of function:

$x = 5$
$x = \pm\sqrt{3}$

Factored form: $f(x) = (x-5)(x-\sqrt{3})(x+\sqrt{3})$

6. Function: $f(x) = x^3 + 3x^2 - 12x - 18$ Factor: $(x-3)$ Roots of function:

$x = -3 + \sqrt{3}$
$x = 3$
~~$x = -3 \pm 3i$~~

$x^2 + 6x + 6$

$x = \dfrac{-6 \pm \sqrt{36 - 4(1)(6)}}{2}$

$$\begin{array}{r|rrrr} +3 & 1 & 3 & -12 & -18 \\ & & 3 & 18 & 18 \\ \hline & 1 & 6 & 6 & \boxed{0} \end{array}$$

$f(x) = (x-3)(x+3-\sqrt{3})(x+3+\sqrt{3})$

Factored form: ~~$f(x) = (x-3)(x-\sqrt{3})(x+\sqrt{3})$~~

7. Function: $f(x) = x^4 - 16$ Factor: $(x-2)$ Roots of function:

$x = 2$
$x = -2$
$x = \pm 2i$

Factored form: $f(x) = (x-2)(x+2)(x-2i)(x+2i)$

mathematics vision project

8. Function: $f(x) = x^3 - x^2 + 4x - 4$

Factor: $(x - 2i)$

Roots of function:

$(x+2i)(x-2i)$

x^2+4

$x = \pm 2i$

$x = 1$

$x - 1$

$x^2+4 \overline{) x^3 - x^2 + 4x - 4}$

$-x^3 \quad -4x$

$-x^2 - 4$

$-x^2 - 4$

0

Factored form: $f(x) = (x-1)(x+2i)(x-2i)$

9. Is it possible for a polynomial with real coefficients to have only one imaginary root? Explain.

Never

10. Based on the Fundamental Theorem of Algebra and the polynomials that you have seen, make a table that shows all the number of roots and the possible combinations of real and imaginary roots for linear, quadratic, cubic, and quartic polynomials.

SET

Topic: Finding the roots and factors of a polynomial

Use the given root to find the remaining roots. Then write the function in factored form.

Function	Roots	Factored form
11. $f(x) = x^3 - 13x^2 + 52x - 60$	$x = 5$ $x=2$ $x=6$	$f(x)=(x-5)(x-2)(x-6)$
12. $g(x) = x^3 + 6x^2 - 11x - 66$	$x = -6$ $x=\pm\sqrt{11}$	$f(x)=(x+6)(x-\sqrt{11})(x+\sqrt{11})$
13. $p(x) = x^3 + 17x^2 + 92x + 150$	$x = -3$ $x=-7\pm i$	$f(x)=$ $(x+3)(x+7-i)(x+7+i)$
14. $q(x) = x^4 - 6x^3 + 3x^2 + 12x - 10$	$x = \sqrt{2}$ $x=-\sqrt{2}$ $x=1$ $x=5$	$q(x)=$ $(x+\sqrt{2})(x-\sqrt{2})(x-1)(x-$