* You can check all your answers w/ the calculator

Double Angle Identities

Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

Your Choice

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Half Angle Identities

Half-Angle Formulas

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$

Can use any of these. I don't use the first one because you have more work to rationalize the deno.

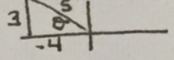
~~tths~~ Want an ∠ on the Unit Circle

Find the following using the double angle identities.

1. Sin(600⁰) $= \sin(2\cdot 300)$
$= 2\sin(300^\circ)\cos(300^\circ)$
$= 2(-\frac{\sqrt{3}}{2})(\frac{1}{2}) = \boxed{-\frac{\sqrt{3}}{2}}$

2. Tan(480⁰) $= \tan(2\cdot 240^\circ)$
$= \frac{2\tan 240^\circ}{1-(\tan 240^\circ)^2} = \frac{2\sqrt{3}}{1-(\sqrt{3})^2}$
$\boxed{= -\sqrt{3}}$

If sinθ = 3/5, π/2 < θ < π, find the exact value of the following. draw △



a. sin(2θ)

$2\sin\theta\cos\theta$

$2(\frac{3}{5})(\frac{-4}{5})$

$= -\frac{24}{25}$

b. cos(2θ)

$\cos^2\theta - \sin^2\theta$

$(-\frac{4}{5})^2 - (\frac{3}{5})^2$

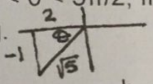
$= \frac{7}{25}$

or $2\cos^2\theta - 1$ either formula

$2(-\frac{4}{5})^2 - 1$

$= \frac{7}{25}$

If tanθ = 1/2, π < θ < 3π/2, find the exact value of the following.



a. sin(2θ)

$= 2\sin\theta\cos\theta$

$= 2(-\frac{1}{\sqrt{5}})(-\frac{2}{\sqrt{5}})$

$= \frac{4}{5}$

b. cos(2θ)

$= \cos^2\theta - \sin^2\theta$

$= (-\frac{2}{\sqrt{5}})^2 - (-\frac{1}{\sqrt{5}})^2$

$= \frac{3}{5}$

*67.5 is half 135

Find the exact value of the following using the half angle identities.

1. cos(67.5⁰) $= \cos\left(\frac{135}{2}\right)$ QI

Check on Calculator

$+\sqrt{\frac{1+\cos 135}{2}} = \sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$\boxed{= \frac{\sqrt{2-\sqrt{2}}}{2}}$

2. sin(165⁰)

$\sin\left(\frac{330}{2}\right)$ QII

$\sqrt{\frac{1-\cos 330}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$

$\boxed{= \frac{\sqrt{2-\sqrt{3}}}{2}}$

3. tan(π/12) $= \tan\left(\frac{\pi/6}{2}\right)$

QI - use any tan formula

$+\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}}$

$\boxed{= \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}}$ * this formula is awful to simplify use another

or $\frac{\sin \pi/6}{1+\cos \pi/6} = \frac{1/2}{1+\frac{\sqrt{3}}{2}}$

$= \frac{1/2}{\frac{2+\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$

$\boxed{= \frac{2-\sqrt{3}}{1}}$

Find the following using half angle identities.

1. cos(22.5⁰) $= \cos\left(\frac{45}{2}\right)$ QI

$+\sqrt{\frac{1+\sqrt{2}/2}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

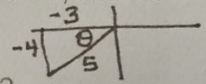
2. sin(5π/12) $\sin\left(\frac{5\pi/6}{2}\right)$

$\sqrt{\frac{1-\left(-\sqrt{3}/2\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

*Check on Calculator

* remember to check your Mode

Given cosα = -3/5 and π < α < 3π/2, find the exact value of the following.



1. cosα/2 QIII

Cos-neg.

$-\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$

$= -\sqrt{\frac{2/5}{2}} = \sqrt{\frac{2}{10}}$

$\frac{\sqrt{2}}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right) = \frac{\sqrt{20}}{10}$

$= \frac{-2\sqrt{5}}{10} = \frac{-\sqrt{5}}{5}$

2. tanα/2

$= \frac{\sin\alpha}{1+\cos\alpha}$ or $\frac{1-\cos\alpha}{\sin\alpha}$

$\frac{-4/5}{1+(-3/5)}$ $\frac{1-(-3/5)}{-4/5}$

$= \frac{-4/5}{2/5}$ $\frac{8/5}{-4/5}$

$= -2$ $= -2$

*

Prove $\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

$= \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}}$

$= \left(\frac{\cos^2\theta - \sin^2\theta}{\cancel{\sin\theta\cos\theta}}\right)\left(\frac{\cancel{\sin\theta\cos\theta}}{\underbrace{\cos^2\theta + \sin^2\theta}_{①}}\right)$

$\cos^2\theta - \sin^2\theta$

$= \cos(2\theta) = \cos(2\theta)$