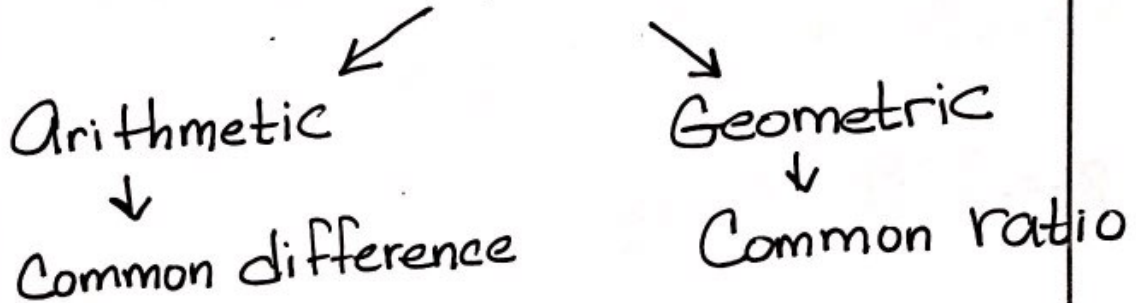

Arithmetic & Geometric Sequences

Explicit Form

*What is a sequence?

pattern of Numbers

*What 2 kinds of sequences do you recall?



Determine if the sequence is arithmetic, geometric, or neither. Find the common difference or ratio.

1) 1, 5, 9, 13, 17, ...

Arithmetic
 $d=4$

$$\begin{aligned} 5-1 &= 4 \\ 9-5 &= 4 \\ 13-9 &= 4 \end{aligned}$$

3) 1, 2, 6, 24, 120, ...

$2/1 = 2$
 $6/2 = 3$
Neither

$$\begin{aligned} 2-1 &= 1 \\ 6-2 &= 4 \end{aligned}$$

2) $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \dots$
 $\frac{1}{4} \div -\frac{1}{2} = -\frac{1}{2}$
 $-\frac{1}{8} \div \frac{1}{4} = -\frac{1}{2}$
 Geometric
 $r = -\frac{1}{2}$

4) $-\frac{1}{4}, \frac{1}{12}, \frac{5}{12}, \frac{3}{4}, \frac{13}{12}, \dots$
 $\frac{1}{12} - -\frac{1}{4} = \frac{1}{3}$
 $\frac{5}{12} - \frac{1}{12} = \frac{1}{3}$
 Arithmetic
 $d = \frac{1}{3}$

Find the Missing Terms

Arithmetic:

24, 31, 38, 45

$$d = \frac{45 - 24}{3} = 7$$

Geometric:

-1, -4, -16, -64

$$r = \frac{-64}{-1} = 64 = \sqrt[3]{64} = 4$$

Arithmetic:

-17, -25, -33, -41, -49

$$d = \frac{-49 - 17}{4} = -8$$

Geometric:

2, 12, 72, 432

$$r = \frac{432}{2} = 216 = \sqrt[3]{216} = 6$$

Explicit Formulas

Arithmetic:

$$a_n = a_1 + d(n - 1)$$

a_n = Value in n^{th} position
 a_1 = 1st term
 d = Common diff
 n = # term (position)

Geometric:

$$a_n = a_1 \cdot r^{n-1}$$

a_1 = 1st term
 r = Common ratio
 n = # term (position)

Write the explicit formula for the following.

1. -3, -7, -11, -15, ... $a_n = -3 + -4(n-1)$

$d = -4$

$a_n = -4n + 1$

2. 1, 1/3, 1/9, ... $a_n = 1 \cdot (1/3)^{n-1}$

$r = 1/3$

$a_n = 1 \cdot (1/3)^{n-1}$

3. -1, 6, 13, 20, ... $a_n = -1 + 7(n-1)$

$d = 7$

$a_n = 7n - 8$

4. 2/3, 1, 3/2, 9/4, ... $a_n = \frac{2}{3} \cdot (3/2)^{n-1}$

$r = 3/2$

$a_n = \frac{2}{3} \cdot (3/2)^{n-1}$

never combine

Find the explicit formula given the following information.

1. Common difference is 2 and $a_{38} = 107$

$a_n = a_1 + d(n-1)$

$107 = a_1 + 74$

$a_1 = 33$

$107 = a_1 + 2(38-1)$

$a_n = 33 + 2(n-1)$

$a_n = 3n + 2n$

$a_n = 2n + 31$

2. Common ratio is 6 and $a_5 = 5184$

$a_n = a_1 \cdot r^{n-1}$

$5184 = a_1 \cdot 6^{5-1}$

$a_n = 4 \cdot (6)^{n-1}$

$5184 = a_1 \cdot 6^4$

$a_1 = 4$

Find the explicit formula given the following information.

1. Arithmetic, $a_{12} = -59$ and $a_{34} = -125$

$$d = \frac{-125 - (-59)}{34 - 12}$$

$$d = -3$$

$$-59 = a_1 + -3(12-1)$$

$$a_1 = -26$$

$$a_n = -26 - 3(n-1)$$

$$\underline{a_n = -3n - 23}$$

2. Geometric, $a_2 = -15$ and $a_5 = -1875$

$$r = \frac{-1875}{-15} = \sqrt[3]{125} = 5$$

$$5 - 2 = 3$$

$$-15 = a_1 (5)^{2-1}$$

$$a_1 = -3$$

$$\underline{a_n = -3(5)^{n-1}}$$

Find the first ~~8~~³ terms of the sequences.

1. $a_n = \frac{2}{n+2}$ $a_1 = \frac{2}{1+2} = \frac{2}{3}$

$$a_2 = \frac{1}{2}$$

$$a_3 = \frac{2}{5}$$

2. $a_n = n^2 - 3n$

$$a_1 = (1)^2 - 3(1) = -2$$

$$a_2 = -2$$

$$a_3 = 0$$

3. $a_n = (n+3)^{n-1}$

$$a_1 = (1+3)^{1-1} = 1$$

$$a_2 = 5$$

$$a_3 = 36$$

Given the sequence, find a pattern and write an explicit equation.

1. 0, 3, 8, 15, 24, ... $a_n = n^2 - 1$
1st 2 3 4 5

2. 2, 6, 12, 20, ... $a_n = n^2 + n$
1 2 3 4

3. 0, 1, 4, 9, ... $a_n = (n-1)^2$
1 2 3 4

An arithmetic sequence has a first term of 8 and a common difference of 4. What term number will have a value of 212?

Recursion: $a_n = a_1 + d(n-1)$ each step of the pattern depends on the previous term.

$$212 = 8 + 4(n-1)$$

$$212 = 8 + 4n - 4$$

$$208 = 4n$$

$$\underline{n = 52}$$