Geometric Series

A geometric series is the expression for the sum of a geometric sequence.

As with arithmetic series, we can use a formula to evaluate geometric finite series.

1st

$$S_n = \frac{a(1 - r^n)}{1 - r} \qquad r \neq 1$$

r = Common ratio

n = # terms

Sum of a Geometric Series

Use the formula to evaluate the series

$3 + 6 + 12 + 24 + 48 + 96.$ $r=2$

$$S = \frac{a(1-r^n)}{1-r}$$

$$= \frac{3(1-2^6)}{1-2} = 189$$

Find the sum of the following series.

1. $a_1 = 60$ $n = 6$ $r = 1/2$

$$S = \frac{60(1-(\frac{1}{2})^6)}{1-\frac{1}{2}} = \underline{118.125}$$

2. Find the sum of: 2, 4, 8, ... to 8 terms

$r=2$

$$S = \frac{2(1-2^8)}{1-2}$$

$$= 510$$

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$E: a_n = a_1 \cdot r^{n-1}$

Evaluate.

$$\sum_{n=1}^{8} 4 \bullet 3^{n-2}$$

$r = 3$

$a_1 = 4 \cdot 3^{1-2}$

$= 4 \cdot 3^{-1} = 4/3$

$$S = \frac{4/3(1-3^8)}{1-3}$$

$= 4373.33$

$$\sum_{k=1}^{6} 3^k \qquad 1 \cdot 3^k$$

$r = 3$

$a_1 = 3^1 = 3$

$$S = \frac{3(1-3^6)}{1-3}$$

$= 1092$

Determine the number of terms in the geometric series.

$$S = \frac{a(1-r^n)}{1-r}$$

$r = 2$ 1. -2 - 4 - 8 - 16 - ....., $S_n = -254$

$$-254 = \frac{-2(1-2^n)}{1-2}$$

$254 = -2(1-2^n)$

$-127 = 1 - 2^n$

$-128 = -2^n$

$128 = 2^n$

$\log_2(128) = n$

$\log_2 2^7 = n$

$7 = n$

2. $a_1 = 2$, $r = 4$, $Sn = 2730$

$$2730 = \frac{2(1-4^n)}{1-4}$$

$-8190 = 2(1-4^n)$

$-4095 = 1 - 4^n$

$-4096 = -4^n$

$4096 = 4^n$

$\log_4 4096 = n$

$n = 6$

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Write the series in summation notation.

1. $\overset{1}{1/2} + \overset{2}{1/3} + \overset{3}{1/4} + \overset{4}{1/5} + \overset{5}{1/6} + \overset{6}{1/7}$

$$\sum_{n=1}^{6} \frac{1}{n+1}$$

2. 2 + 6 + 18 + 54 + 162 G

$\frac{6}{2}$ $\frac{18}{6}$ $\frac{54}{18} = r = 3$

$$\sum_{i=1}^{5} 2 \cdot 3^{n-1}$$

$a_n = a_1 \cdot r^{n-1}$

3. $1 + 5 + \overset{9}{\cancel{8}} + 13 + \overset{17}{\cancel{20}} + \overset{21}{\cancel{29}} + \overset{24}{\cancel{40}}$

$$\sum_{i=1}^{7} 4n-3$$

$a_n = a_1 + d(n-1)$

$a_n = 1 + 4(n-1)$

$4n-3$

In some cases you can evaluate an infinite geometric series.

$$S = \frac{a_1}{1 - r}$$

When $|r| < 1$, the series Converges, or gets closer and closer to the sum.

When $|r| > 1$, the series Diverges, or approaches no limit.

Decide whether or not each geometric series converges or diverges. State whether the series has a sum.

1. 1 - 1/3 + 1/9 - ...

$S = \frac{a}{1-r}$

$r = -\frac{1}{3}$ $\left|-\frac{1}{3}\right| < 1$

Converge $S = \frac{1}{1 - -\frac{1}{3}} = \frac{3}{4}$

2. 1 + 1/5 + 1/25 + ...

$r = \frac{1}{5}$ Converges $S = \frac{1}{1 - \frac{1}{5}} = \frac{5}{4}$

3. 4 + 8 + 16 + ...

diverges

$r = 2$ $r > 1$

No Sum!

Decide whether or not each geometric series converges or diverges. Find the sum if able.

1. 2 + 2/3 + 2/9 + ... Converges

$S = \frac{3}{2}$

2. 1/10 - 1/5 + 2/5 - ... diverges

no sum

3. 4 - 2 + 1 - 1/2 + ... Converges $S = \frac{2}{3}$