3.4 EXERCISES

In Problems 1–12, find the domain of each rational function.

1. $R(x) = \dfrac{4x}{x-3}$

2. $R(x) = \dfrac{5x^2}{3+x}$

3. $H(x) = \dfrac{-4x^2}{(x-2)(x+4)}$

4. $G(x) = \dfrac{6}{(x+3)(4-x)}$

5. $F(x) = \dfrac{3x(x-1)}{2x^2-5x-3}$

6. $Q(x) = \dfrac{-x(1-x)}{3x^2+5x-2}$

7. $R(x) = \dfrac{x}{x^3-8}$

8. $R(x) = \dfrac{x}{x^4-1}$

9. $H(x) = \dfrac{3x^2+x}{x^2+4}$

10. $G(x) = \dfrac{x-3}{x^4+1}$

11. $R(x) = \dfrac{3(x^2-x-6)}{4(x^2-9)}$

12. $F(x) = \dfrac{-2(x^2-4)}{3(x^2+4x+4)}$

In Problems 13–22, use the graph shown to find:

(a) The domain and range of each function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any

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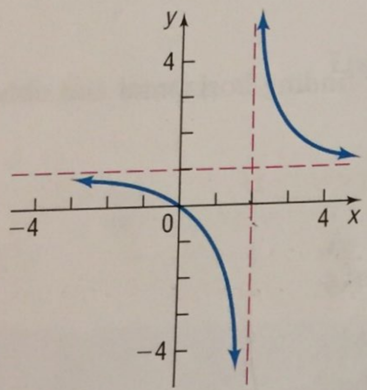

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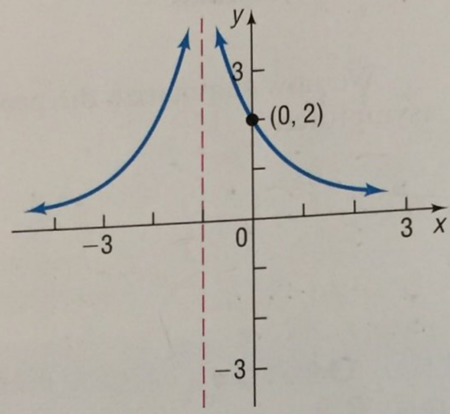

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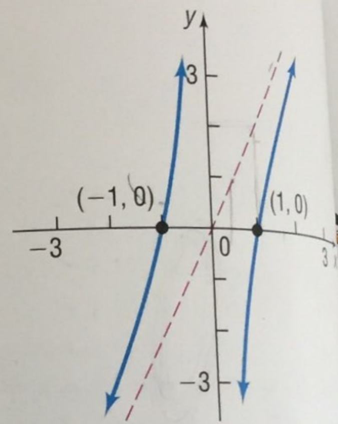

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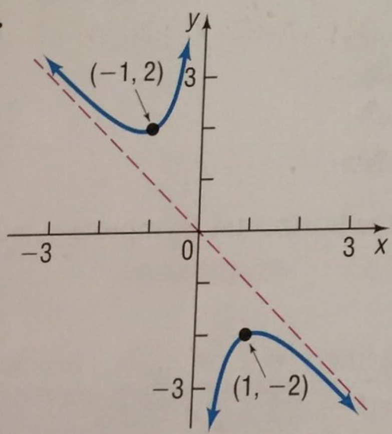

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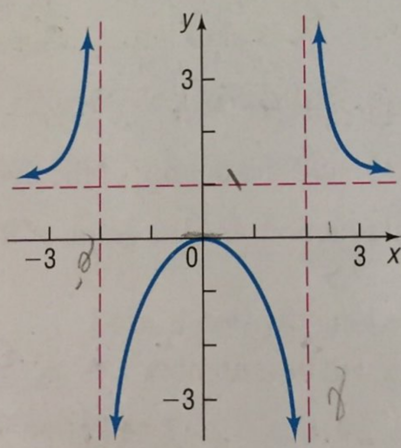

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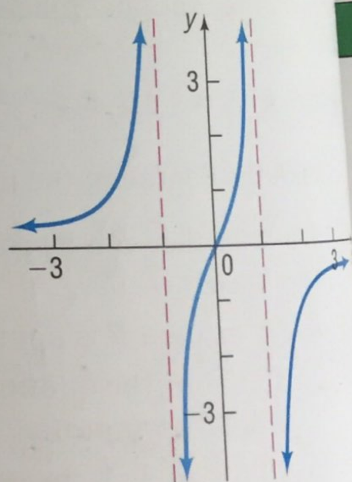

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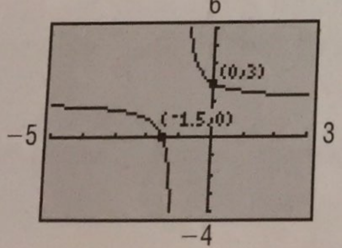

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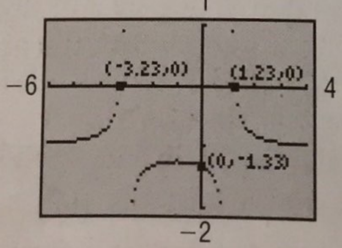

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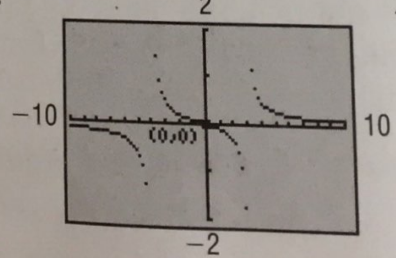

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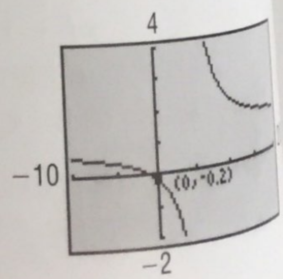

In Proble
graphing

23. R(x)

26. G(x

29. F(

32. R(

In Pro
Verify

35. F

38.

41.

44.

47

In Problems 23–34, show the steps required to graph each rational function using transformations. Verify your results using a graphing utility.

23. $R(x) = \dfrac{1}{(x-1)^2}$
24. $R(x) = \dfrac{3}{x}$
25. $H(x) = \dfrac{-2}{x+1}$
26. $G(x) = \dfrac{2}{(x+2)^2}$
27. $R(x) = \dfrac{1}{x^2+4x+4}$
28. $R(x) = \dfrac{1}{x-1} + 1$
29. $F(x) = 1 - \dfrac{1}{x}$
30. $Q(x) = 1 + \dfrac{1}{x}$
31. $R(x) = \dfrac{x^2-4}{x^2}$
32. $R(x) = \dfrac{x-4}{x}$
33. $G(x) = 1 + \dfrac{2}{(x-3)^2}$
34. $F(x) = 2 - \dfrac{1}{x+1}$

In Problems 35–46, find the vertical, horizontal, and oblique asymptotes, if any, of each rational function without graphing. Verify your results using a graphing utility.

35. $R(x) = \dfrac{3x}{x+4}$
36. $R(x) = \dfrac{3x+5}{x-6}$
37. $H(x) = \dfrac{x^4+2x^2+1}{x^2-x+1}$
38. $G(x) = \dfrac{-x^2+1}{x+5}$
39. $T(x) = \dfrac{x^3}{x^4-1}$
40. $P(x) = \dfrac{4x^5}{x^3-1}$
41. $Q(x) = \dfrac{5-x^2}{3x^4}$
42. $F(x) = \dfrac{-2x^2+1}{2x^3+4x^2}$
43. $R(x) = \dfrac{3x^4+4}{x^3+3x}$
44. $R(x) = \dfrac{6x^2+x+12}{3x^2-5x-2}$
45. $G(x) = \dfrac{x^3-1}{x-x^2}$ $-x(x-1)$
46. $F(x) = \dfrac{x-1}{x-x^3}$

47. If the graph of a rational function R has the vertical asymptote $x = 4$, then the factor $x - 4$ must be present in the denominator of R. Explain why.
48. If the graph of a rational function R has the horizontal asymptote $y = 2$, then the degree of the numerator of R equals the degree of the denominator of R. Explain why.
49. Can the graph of a rational function have both a horizontal and an oblique asymptote? Explain.
50. Make up a rational function that has $y = 2x + 1$ as an oblique asymptote. Explain the methodology that you used.

PREPARING FOR THIS SECTION

Before getting started, review the following concepts:

- ✓ Intercepts (pp. 16–18)
- ✓ Symmetry (pp. 18–19)
- ✓ Even and Odd Functions (pp. 122–124)

3.5 RATIONAL FUNCTIONS II: ANALYZING GRAPHS

1 Analyze the Graph of a Rational Function