

7.3 EXERCISES

In Problems 1–12, use the information given about the angle θ , $0 \leq \theta \leq 2\pi$, to find the exact value of:

(a) $\sin(2\theta)$

(b) $\cos(2\theta)$

(c) $\sin \frac{\theta}{2}$

(d) $\cos \frac{\theta}{2}$

1. $\sin \theta = \frac{3}{5}$, $0 < \theta < \pi/2$

2. $\cos \theta = \frac{3}{5}$, $0 < \theta < \pi/2$

3. $\tan \theta = \frac{4}{3}$, $\pi < \theta < 3\pi/2$

4. $\tan \theta = \frac{1}{2}$, $\pi < \theta < 3\pi/2$

5. $\cos \theta = -\sqrt{6}/3$, $\pi/2 < \theta < \pi$

6. $\sin \theta = -\sqrt{3}/3$, $3\pi/2 < \theta < 2\pi$

7. $\sec \theta = 3$, $\sin \theta > 0$

8. $\csc \theta = -\sqrt{5}$, $\cos \theta < 0$

9. $\cot \theta = -2$, $\sec \theta < 0$

10. $\sec \theta = 2$, $\csc \theta < 0$

11. $\tan \theta = -3$, $\sin \theta < 0$

12. $\cot \theta = 3$, $\cos \theta < 0$

In Problems 13–22, use the half-angle formulas to find the exact value of each trigonometric function.

13. $\sin 22.5^\circ$

14. $\cos 22.5^\circ$

15. $\tan \frac{7\pi}{8}$

16. $\tan \frac{9\pi}{8}$

17. $\cos 165^\circ$

18. $\sin 195^\circ$

19. $\sec \frac{15\pi}{8}$

20. $\csc \frac{7\pi}{8}$

21. $\sin\left(-\frac{\pi}{8}\right)$

22. $\cos\left(-\frac{3\pi}{8}\right)$

23. Show that $\sin^4 \theta = \frac{3}{8} - \frac{1}{2} \cos(2\theta) + \frac{1}{8} \cos(4\theta)$.

25. Show that $\sin(4\theta) = (\cos \theta)(4 \sin \theta - 8 \sin^3 \theta)$.

27. Find an expression for $\sin(5\theta)$ as a fifth-degree polynomial in the variable $\sin \theta$.

24. Develop a formula for $\cos(3\theta)$ as a third-degree polynomial in the variable $\cos \theta$.

26. Develop a formula for $\cos(4\theta)$ as a fourth-degree polynomial in the variable $\cos \theta$.

28. Find an expression for $\cos(5\theta)$ as a fifth-degree polynomial in the variable $\cos \theta$.

In Problems 29–48, establish each identity.

29. $\cos^4 \theta - \sin^4 \theta = \cos(2\theta)$

30. $\frac{\cot \theta - \tan \theta}{\cot \theta + \tan \theta} = \cos(2\theta)$

31. $\cot(2\theta) = \frac{\cot^2 \theta - 1}{2 \cot \theta}$

32. $\cot(2\theta) = \frac{1}{2}(\cot \theta - \tan \theta)$

33. $\sec(2\theta) = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

34. $\csc(2\theta) = \frac{1}{2} \sec \theta \csc \theta$

35. $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$

36. $(4 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta) = \sin(4\theta)$

37. $\frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cot \theta - 1}{\cot \theta + 1}$

38. $\sin^2 \theta \cos^2 \theta = \frac{1}{8}[1 - \cos(4\theta)]$

39. $\sec^2 \frac{\theta}{2} = \frac{2}{1 + \cos \theta}$

40. $\csc^2 \frac{\theta}{2} = \frac{2}{1 - \cos \theta}$

41. $\cot^2 \frac{\theta}{2} = \frac{\sec \theta + 1}{\sec \theta - 1}$

42. $\tan \frac{\theta}{2} = \csc \theta - \cot \theta$

43. $\cos \theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)}$

44. $1 - \frac{1}{2} \sin(2\theta) = \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta}$

45. $\frac{\sin(3\theta)}{\sin \theta} - \frac{\cos(3\theta)}{\cos \theta} = 2$

46. $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = 2 \tan(2\theta)$

47. $\tan(3\theta) = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

48. $\tan \theta + \tan(\theta + 120^\circ) + \tan(\theta + 240^\circ) = 3 \tan(3\theta)$