Figure 33

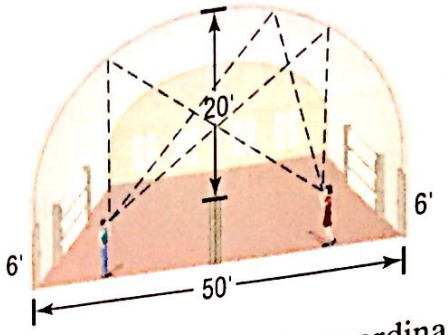
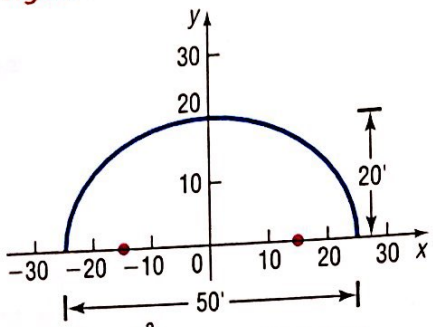


Figure 34



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a = 25, b = 20$

Solution We set up a rectangular coordinate system so that the center of the ellips at the origin and the major axis is along the x-axis. See Figure 34. The eq tion of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $a = 25$ and $b = 20$. Since

$$c^2 = a^2 - b^2 = 25^2 - 20^2 = 625 - 400 = 225$$

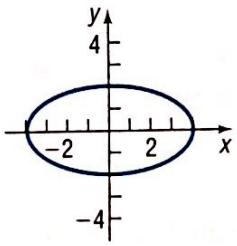
we have $c = 15$. Thus, the foci are located 15 feet from the center of the lipse along the major axis.

10.3 EXERCISES

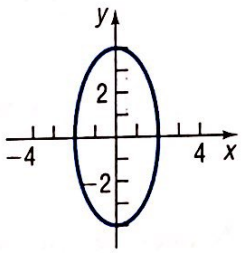
In Problems 1–4, the graph of an ellipse is given. Match each graph to its equation.

A. $\frac{x^2}{4} + y^2 = 1$ B. $x^2 + \frac{y^2}{4} = 1$ C. $\frac{x^2}{16} + \frac{y^2}{4} = 1$ D. $\frac{x^2}{4} + \frac{y^2}{16} = 1$

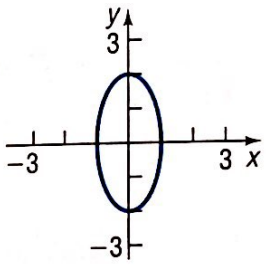
1.



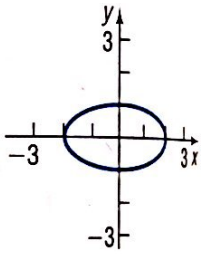
2.



3.



4.

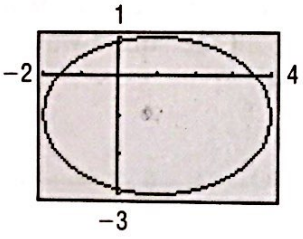


In Problems 5–8, the graph of an ellipse is given. Match each graph to its equation.

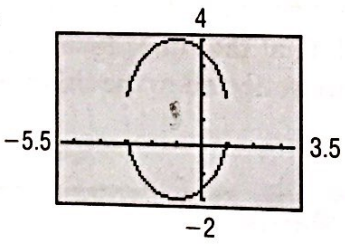
A. $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{9} = 1$ B. $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$

C. $\frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1$ D. $\frac{(x+1)^2}{9} + \frac{(y-1)^2}{4} = 1$

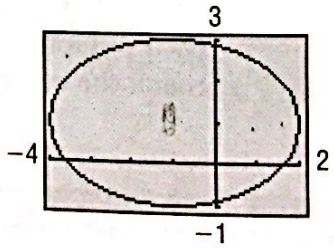
5.



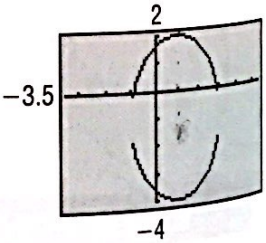
6.



7.



8.



In Problems 9–18, find the vertices and foci of each ellipse. Graph each equation using a graphing utility.

9. $\frac{x^2}{25} + \frac{y^2}{4} = 1$

10. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

11. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

12. $x^2 + \frac{y^2}{16} = 1$

13. $4x^2 + y^2 = 16$

14. $x^2 + 9y^2 = 18$

15. $4y^2 + x^2 = 8$

16. $4y^2 + 9x^2 = 36$

17. $x^2 + y^2 = 16$

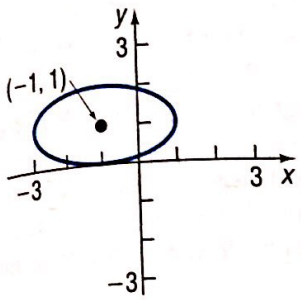
18. $x^2 + y^2 = 4$

In Problems 19–28, find an equation for each ellipse. Graph the equation by hand.

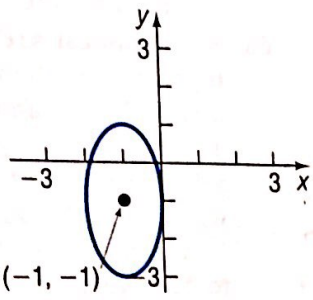
19. Center at $(0, 0)$; focus at $(3, 0)$; vertex at $(5, 0)$
20. Center at $(0, 0)$; focus at $(-1, 0)$; vertex at $(3, 0)$
21. Center at $(0, 0)$; focus at $(0, -4)$; vertex at $(0, 5)$
22. Center at $(0, 0)$; focus at $(0, 1)$; vertex at $(0, -2)$
23. Foci at $(\pm 2, 0)$; length of the major axis is 6
24. Focus at $(0, -4)$; vertices at $(0, \pm 8)$
25. Foci at $(0, \pm 3)$; x-intercepts are ± 2
26. Foci at $(0, \pm 2)$; length of the major axis is 8
27. Center at $(0, 0)$; vertex at $(0, 4)$; $b = 1$
28. Vertices at $(\pm 5, 0)$; $c = 2$

In Problems 29–32, write an equation for each ellipse.

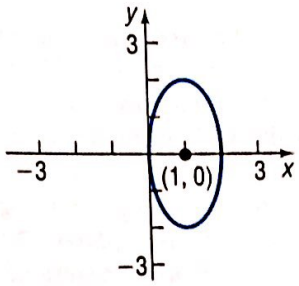
29.



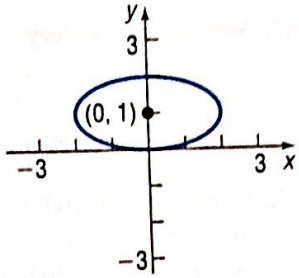
30.



31.



32.



In Problems 33–44, find the center, foci, and vertices of each ellipse. Graph each equation using a graphing utility.

33. $\dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{9} = 1$
34. $\dfrac{(x+4)^2}{9} + \dfrac{(y+2)^2}{4} = 1$
35. $(x+5)^2 + 4(y-4)^2 = 16$
36. $9(x-3)^2 + (y+2)^2 = 18$
37. $x^2 + 4x + 4y^2 - 8y + 4 = 0$
38. $x^2 + 3y^2 - 12y + 9 = 0$
39. $2x^2 + 3y^2 - 8x + 6y + 5 = 0$
40. $4x^2 + 3y^2 + 8x - 6y = 5$
41. $9x^2 + 4y^2 - 18x + 16y - 11 = 0$
42. $x^2 + 9y^2 + 6x - 18y + 9 = 0$
43. $4x^2 + y^2 + 4y = 0$
44. $9x^2 + y^2 - 18x = 0$

In Problems 45–54, find an equation for each ellipse. Graph the equation by hand.

45. Center at $(2, -2)$; vertex at $(7, -2)$; focus at $(4, -2)$
46. Center at $(-3, 1)$; vertex at $(-3, 3)$; focus at $(-3, 0)$
47. Vertices at $(4, 3)$ and $(4, 9)$; focus at $(4, 8)$
48. Foci at $(1, 2)$ and $(-3, 2)$; vertex at $(-4, 2)$
49. Foci at $(5, 1)$ and $(-1, 1)$; length of the major axis is 8
50. Vertices at $(2, 5)$ and $(2, -1)$; $c = 2$
51. Center at $(1, 2)$; focus at $(4, 2)$; contains the point $(1, 3)$
52. Center at $(1, 2)$; focus at $(1, 4)$; contains the point $(2, 2)$
53. Center at $(1, 2)$; vertex at $(4, 2)$; contains the point $(1, 3)$
54. Center at $(1, 2)$; vertex at $(1, 4)$; contains the point $(2, 2)$

In Problems 55–58, graph each function by hand. Use a graphing utility to verify your results.
[**Hint:** Notice that each function is half an ellipse.]

55. $f(x) = \sqrt{16 - 4x^2}$
56. $f(x) = \sqrt{9 - 9x^2}$
57. $f(x) = -\sqrt{64 - 16x^2}$
58. $f(x) = -\sqrt{4 - 4x^2}$

59. **Semielliptical Arch Bridge** An arch in the shape of the upper half of an ellipse is used to support a bridge that is to span a river 20 meters wide. The center of the arch is 6 meters above the center of the river (see the figure). Write an equation for the ellipse in which the x-axis coincides with the water level and the y-axis passes through the center of the arch.