In Problems 15–24, find the center (h, k) and radius r of each circle. By hand, graph each circle.

15. $x^2 + y^2 = 4$

16. $x^2 + (y - 1)^2 = 1$

17. $(x - 3)^2 + y^2 = 4$

18. $(x + 1)^2 + (y - 1)^2 = 2$

19. $x^2 + y^2 + 4x - 4y - 1 = 0$

20. $x^2 + y^2 - 6x + 2y + 9 = 0$

21. $x^2 + y^2 - x + 2y + 1 = 0$

22. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

23. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

24. $2x^2 + 2y^2 + 8x + 7 = 0$

In Problems 25–30, find the general form of the equation of each circle.

25. Center at the origin and containing the point $(-2, 3)$

26. Center $(1, 0)$ and containing the point $(-3, 2)$

27. Center $(2, 3)$ and tangent to the x-axis

28. Center $(-3, 1)$ and tangent to the y-axis

29. With endpoints of a diameter at $(1, 4)$ and $(-3, 2)$

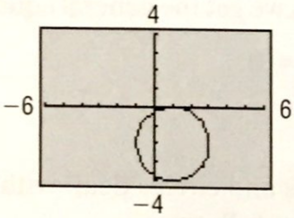
30. With endpoints of a diameter at $(4, 3)$ and $(0, 1)$

In Problems 31–34, match each graph with the correct equation.

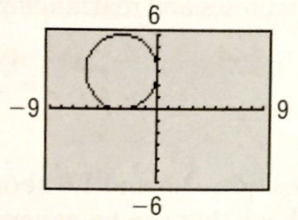
(a) $(x - 3)^2 + (y + 3)^2 = 9$ (b) $(x + 1)^2 + (y - 2)^2 = 4$

(c) $(x - 1)^2 + (y + 2)^2 = 4$ (d) $(x + 3)^2 + (y - 3)^2 = 9$

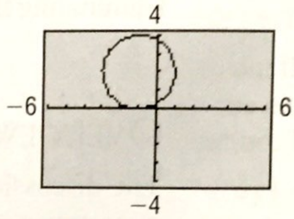
31.



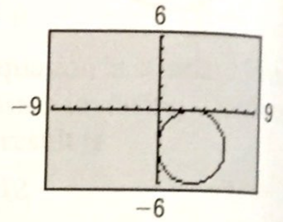
32.



33.

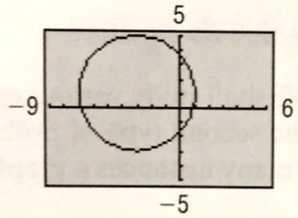


34.

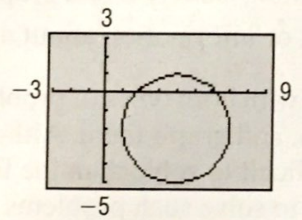


In Problems 35–38, find the standard form of the equation of each circle.

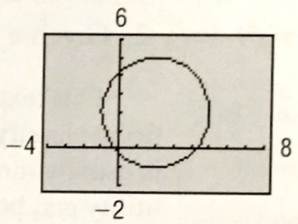
35.



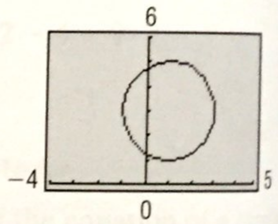
36.



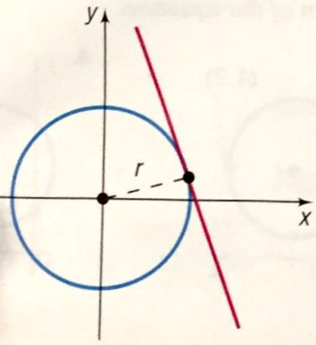
37.



38.



39. The **tangent line** to a circle may be defined as the line that intersects the circle in a single point, called the **point of tangency** (see the figure).



the equation of the circle is $x^2 + y^2 = r^2$ and the equation of the tangent line is $y = mx + b$, show that:

(a) $r^2(1 + m^2) = b^2$
[**Hint:** The quadratic equation $x^2 + (mx + b)^2 = r^2$ has exactly one solution.]

(b) The point of tangency is $(-r^2 m/b, r^2/b)$.

(c) The tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

40. The Greek Method for Finding Tangents The Greek method for finding the equation of the tangent line to a circle used the fact that at any point on a circle the lines containing the center and the tangent line are perpendicular (see Problem 39). Use this method to find an equation of the tangent line to the circle $x^2 + y^2 = 9$ at the point $(1, 2\sqrt{2})$.

41. Use the Greek method described in Problem 40 to find an equation of the tangent line to the circle $x^2 + y^2 - 4x + 6y + 4 = 0$ at the point $(3, 2\sqrt{2} - 3)$.

42. Refer to Problem 39. The line $x - 2y = -4$ is tangent to a circle at $(0, 2)$. The line $y = 2x - 7$ is tangent to the same circle at $(3, -1)$. Find the center of the circle.

43. Find an equation of the line containing the centers of the two circles

$$x^2 + y^2 - 4x + 6y + 4 = 0$$

and

$$x^2 + y^2 + 6x + 4y + 9 = 0$$

44. If a circle of radius 2 is made to roll along the x-axis, what is an equation for the path of the center of the circle?