(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) Find the cubic function of best fit.
(c) Draw the cubic function of best fit on the scatter diagram.
(d) Use the function found in part (b) to predict the average fuel consumption for cars in 1999 ($x = 16$).
(e) Do you think the function found in part (b) will be useful in predicting the average fuel consumption for cars in 2010? Why?

Solution (a) Figure 36 shows the scatter diagram. A cubic relation with $a > 0$ may exist between the two variables.

(b) Upon executing the CUBIC REGression program, we obtain the results shown in Figure 37. The output that the utility provides shows us the equation $y = ax^3 + bx^2 + cx + d$. The cubic function of best fit is

$$C(x) = 0.398x^3 - 6.793x^2 + 26.313x + 476.692$$

where x represents the year and C represents the average fuel consumption.

(c) Figure 38 shows the graph of the cubic function of best fit on the scatter diagram. The function seems to fit the data well.

Figure 36

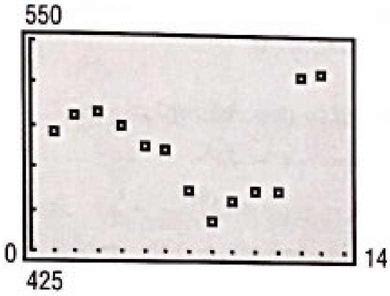


Figure 37

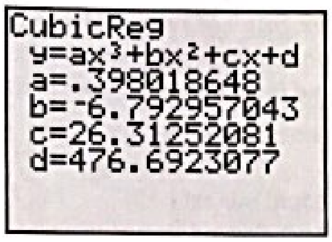
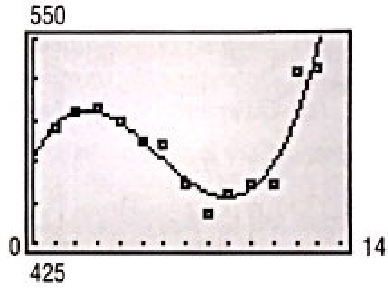


Figure 38



(d) We evaluate the function $C(x)$ for $x = 16$.

$$C(x) = 0.398(16)^3 - 6.793(16)^2 + 26.313(16) + 476.692 \approx 789$$

So we predict that the average annual fuel consumption for cars will be 789 gallons in 1999.

(e) The graph of the function will continually increase for years beyond 1996, indicating that the average fuel consumption for cars will continue to increase. While it is likely that people will continue to drive more in the years to come, the increases will probably start to subside, since cars are getting better gas mileage. So the function probably will not be useful in predicting the average fuel consumption per car in 2010. ■

3.3 EXERCISES

In Problems 1–10, determine which functions are polynomial functions. For those that are, state the degree. For those that are not, tell why not.

1. $f(x) = 4x + x^3$ **2.** $f(x) = 5x^2 + 4x^4$ **3.** $g(x) = \frac{1 - x^2}{2}$

4. $h(x) = 3 - \frac{1}{2}x$ **5.** $f(x) = 1 - \frac{1}{x}$ **6.** $f(x) = x(x - 1)$

7. $g(x) = x^{3/2} - x^2 + 2$

8. $h(x) = \sqrt{x}(\sqrt{x} - 1)$

9. $F(x) = 5x^4 - \pi x^3 + \frac{1}{2}$

10. $F(x) = \frac{x^2 - 5}{x^3}$

In Problems 11–16, form a polynomial whose zeros and degree are given.

11. Zeros: −1, 1, 3; degree 3

12. Zeros: −2, 2, 3; degree 3

13. Zeros: −3, 0, 4; degree 3

14. Zeros: −4, 0, 2; degree 3

15. Zeros: −4, −1, 2, 3; degree 4

16. Zeros: −3, −1, 2, 5; degree 4

In Problems 17–26, for each polynomial function, list each real zero and its multiplicity. Determine whether the graph crosses or touches the x-axis at each x-intercept. Do not graph f.

17. $f(x) = 3(x - 7)(x + 3)^2$

18. $f(x) = 4(x + 4)(x + 3)^3$

19. $f(x) = 4(x^2 + 1)(x - 2)^3$

20. $f(x) = 2(x - 3)(x + 4)^3$

21. $f(x) = -2\left(x + \frac{1}{2}\right)^2(x^2 + 4)^2$

22. $f(x) = \left(x - \frac{1}{3}\right)^2(x - 1)^3$

23. $f(x) = (x - 5)^3(x + 4)^2$

24. $f(x) = (x + \sqrt{3})^2(x - 2)^4$

25. $f(x) = 3(x^2 + 8)(x^2 + 9)^2$

26. $f(x) = -2(x^2 + 3)^3$

In Problems 27–62, for each polynomial function f:

(a) Using a graphing utility, graph f.
(b) Find the x- and y-intercepts.
(c) Determine whether each x-intercept is of odd or even multiplicity.
(d) Find the power function that the graph of f resembles for large values of |x|.
(e) Determine the number of turning points on the graph of f.
(f) Determine the local maxima and local minima, if any exist, rounded to two decimal places.

27. $f(x) = (x - 1)^2$

28. $f(x) = (x - 2)^3$

29. $f(x) = x^2(x - 3)$

30. $f(x) = x(x + 2)^2$

31. $f(x) = 6x^3(x + 4)$

32. $f(x) = 5x(x - 1)^3$

33. $f(x) = -4x^2(x + 2)$

34. $f(x) = -\frac{1}{2}x^3(x + 4)$

35. $f(x) = x(x - 2)(x + 4)$

36. $f(x) = x(x + 4)(x - 3)$

37. $f(x) = 4x - x^3$

38. $f(x) = x - x^3$

39. $f(x) = x^2(x - 2)(x + 2)$

40. $f(x) = x^2(x - 3)(x + 4)$

41. $f(x) = x^2(x - 2)^2$

42. $f(x) = x^3(x - 3)$

43. $f(x) = x^2(x - 3)(x + 1)$

44. $f(x) = x^2(x - 3)(x - 1)$

45. $f(x) = x(x + 2)(x - 4)(x - 6)$

46. $f(x) = x(x - 2)(x + 2)(x + 4)$

47. $f(x) = x^2(x - 2)(x^2 + 3)$

48. $f(x) = x^2(x^2 + 1)(x + 4)$

49. $f(x) = x^3 + 0.2x^2 - 1.5876x - 0.31752$

50. $f(x) = x^3 - 0.8x^2 - 4.6656x + 3.73248$

51. $f(x) = x^3 + 2.56x^2 - 3.31x + 0.89$

52. $f(x) = x^3 - 2.91x^2 - 7.668x - 3.8151$

53. $f(x) = x^4 - 2.5x^2 + 0.5625$

54. $f(x) = x^4 - 18.5x^2 + 50.2619$

55. $f(x) = x^4 + 0.65x^3 - 16.6319x^2 + 14.209335x - 3.1264785$

56. $f(x) = x^4 + 3.45x^3 - 11.6639x^2 - 5.864241x - 0.69257738$

57. $f(x) = \pi x^3 + \sqrt{2}x^2 - x - 2$

58. $f(x) = -2x^3 + \pi x^2 + \sqrt{3}x + 1$

59. $f(x) = 2x^4 - \pi x^3 + \sqrt{5}x - 4$

60. $f(x) = -1.2x^4 + 0.5x^2 - \sqrt{3}x + 2$

61. $f(x) = -2x^5 - \sqrt{2}x^2 - x - \sqrt{2}$

62. $f(x) = \pi x^5 + \pi x^4 + \sqrt{3}x + 1$

63. Co
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64. C
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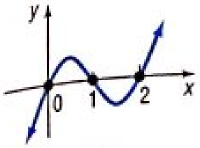
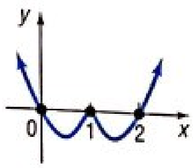
n

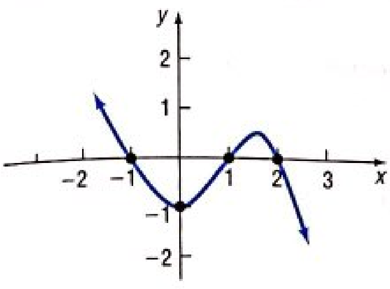
63. Consult illustrations (a)–(d). Construct a polynomial function that might have this graph. (More than one answer may be possible.)



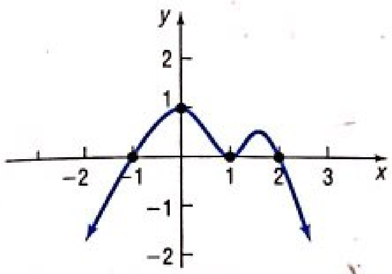
(a)



(b)



(c)



(d)

64. **Cost of Printing** The following data represent the weekly cost of printing textbooks C (in thousands) and the number x of texts printed (in thousands).

Number of Text Books, x	Cost, C
0	100
5	128.1
10	144
13	153.5
17	161.2
18	162.6
20	166.3
23	178.9
25	190.2
27	221.8

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) Find the average rate of change of cost from 10,000 to 13,000 textbooks.
(c) What is the average rate of change in the cost of producing from 18,000 to 20,000 textbooks?
(d) Find the cubic function of best fit to the data.
(e) Graph the cubic function of best fit on the scatter diagram.
(f) Use the function found in (d) to predict the cost of printing 22,000 texts per week.
(g) Interpret the y-intercept.

65. **Motor Vehicle Thefts** The following data represent the number of motor vehicle thefts (in thousands) in the United States for the years 1987–1997, where 1 represents 1987, 2 represents 1988, and so on.

Year, x	Motor Vehicle Thefts, M
1987, 1	1289
1988, 2	1433
1989, 3	1565
1990, 4	1636
1991, 5	1662
1992, 6	1611
1993, 7	1563
1994, 8	1539
1995, 9	1472
1996, 10	1394
1997, 11	1354

Source: U.S. Federal Bureau of Investigation

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) Find the cubic function of best fit.
(c) Graph the cubic function of best fit on the scatter diagram.
(d) Use the function found in (b) to predict the number of motor vehicle thefts in 1998.
(e) Check the prediction of part (d) against actual data. Do you think that the function given in part (b) will be useful in predicting the number of motor vehicle thefts in 1999?

66. **Larceny Thefts** The following data represent the number of larceny thefts (in thousands) in the United States for the years 1983–1993, where 1 represents 1983, 2 represents 1984, and so on.

Year, x	Larceny Thefts, L
1983, 1	6713
1984, 2	6592
1985, 3	6926
1986, 4	7257
1987, 5	7500
1988, 6	7706
1989, 7	7872
1990, 8	7946
1991, 9	8142
1992, 10	7915
1993, 11	7821

Source: U.S. Federal Bureau of Investigation