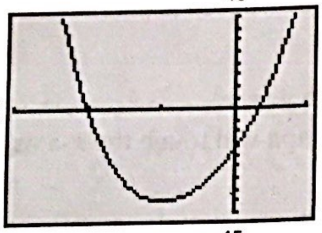
STEP 3: The Rational Zeros Theorem provides information about the potential rational zeros of polynomials with integer coefficients. For this polynomial (which has integer coefficients), the potential rational zeros are

$$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$$

STEP 4: Figure 14 shows the graph of f. The graph has the characteristics that we expect of this polynomial of degree 4: It behaves like $y =$ for large $|x|$ and has y-intercept -18. There are x-intercepts near and between 0 and 1.

Figure 14



STEP 5: Since $f(-2) = 0$, we know that -2 is a zero of f. We use synthetic division to factor f.

$-2)$	3	5	25	45	-18
		-6	2	-54	18
	3	-1	27	-9	0

So $f(x) = (x + 2)(3x^3 - x^2 + 27x - 9)$. From the graph of f and the list of potential rational zeros, it appears that 1/3 may be a zero of f. Since $f\left(\frac{1}{3}\right) = 0$, we know that $\frac{1}{3}$ is a zero of f. We use synthetic division on the depressed equation of f to factor.

$1/3)$	3	-1	27	-9
		1	0	9
	3	0	27	0

Using the bottom row of the synthetic division, we find

$$f(x) = (x + 2)\left(x - \frac{1}{3}\right)(3x^2 + 27) = 3(x + 2)\left(x - \frac{1}{3}\right)(x^2 +$$

The factor $x^2 + 9$ does not have any real zeros; its complex zeros are $\pm 3i$. The complex zeros of $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x -$ are $-2, 1/3, 3i, -3i$.

NOW WORK PROBLEM 27.

4.3 EXERCISES

In Problems 1–10, information is given about a polynomial $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f.

1. Degree 3; zeros: $3, 4 - i$
2. Degree 3; zeros: $4, 3 + i$
3. Degree 4; zeros: $i, 1 + i$
4. Degree 4; zeros: $1, 2, 2 + i$
5. Degree 5; zeros: $1, i, 2i$
6. Degree 5; zeros: $0, 1, 2, i$
7. Degree 4; zeros: $i, 2, -2$
8. Degree 4; zeros: $2 - i, -i$
9. Degree 6; zeros: $2, 2 + i, -3 - i, 0$
10. Degree 6; zeros: $i, 3 - 2i, -2 + i$

In Problems 11–16, form a polynomial $f(x)$ with real coefficients having the given degree and zeros.

11. Degree 4; zeros: $3 + 2i$; 4, multiplicity 2
12. Degree 4; zeros: $i, 1 + 2i$
13. Degree 5; zeros: 2, multiplicity 1; $-i$; $1 + i$
14. Degree 6; zeros: $i, 4 - i$; $2 + i$
15. Degree 4; zeros: 3, multiplicity 2; $-i$
16. Degree 5; zeros: 1, multiplicity 3; $1 + i$

In Problems 17–24, use the given zero to find the remaining zeros of each function. Graph the function to verify your results.

17. $f(x) = x^3 - 4x^2 + 4x - 16$; zero: $2i$

18. $g(x) = x^3 + 3x^2 + 25x + 75$; zero: $-5i$

19. $f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$; zero: $-2i$

20. $h(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$; zero: $3i$

21. $h(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$; zero: $3 - 2i$

22. $f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$; zero: $1 + 3i$

23. $h(x) = 3x^5 + 2x^4 + 15x^3 + 10x^2 - 528x - 352$; zero: $-4i$

24. $g(x) = 2x^5 - 3x^4 - 5x^3 - 15x^2 - 207x + 108$; zero: $3i$

In Problems 25–34, find the complex zeros of each polynomial function. Check your results by evaluating the function at each of the zeros.

25. $f(x) = x^3 - 1$

26. $f(x) = x^4 - 1$

27. $f(x) = x^3 - 8x^2 + 25x - 26$

28. $f(x) = x^3 + 13x^2 + 57x + 85$

29. $f(x) = x^4 + 5x^2 + 4$

30. $f(x) = x^4 + 13x^2 + 36$

31. $f(x) = x^4 + 2x^3 + 22x^2 + 50x - 75$

32. $f(x) = x^4 + 3x^3 - 19x^2 + 27x - 252$

33. $f(x) = 3x^4 - x^3 - 9x^2 + 159x - 52$

34. $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65$

In Problems 35 and 36, tell why the facts given are contradictory.

35. $f(x)$ is a polynomial of degree 3 whose coefficients are real numbers; its zeros are $4 + i$, $4 - i$, and $2 + i$.

36. $f(x)$ is a polynomial of degree 3 whose coefficients are real numbers; its zeros are 2, i, and $3 + i$.

37. $f(x)$ is a polynomial of degree 4 whose coefficients are real numbers; three of its zeros are 2, $1 + 2i$, and $1 - 2i$. Explain why the remaining zero must be a real number.

38. $f(x)$ is a polynomial of degree 4 whose coefficients are real numbers; two of its zeros are -3 and $4 - i$. Explain why one of the remaining zeros must be a real number. Write down one of the missing zeros.

PREPARING FOR THIS SECTION

Before getting started, review the following:

✓ Solving Inequalities (Section 1.5)

4.4 POLYNOMIAL AND RATIONAL INEQUALITIES

OBJECTIVES

1. Solve Polynomial Inequalities Graphically and Algebraically
2. Solve Rational Inequalities Graphically and Algebraically

1 In this section we consider inequalities that involve polynomials of degree 2 and higher, as well as some that involve rational expressions. We will solve these inequalities using both a graphing approach and an algebraic approach.

To solve polynomial and rational inequalities algebraically, we follow these steps:

POLYNOMIAL