STEP 2: We solve for y to get the explicit form of the inverse. Since $y \geq 0$, only one solution for y is obtained.

$$y = \sqrt{x}$$

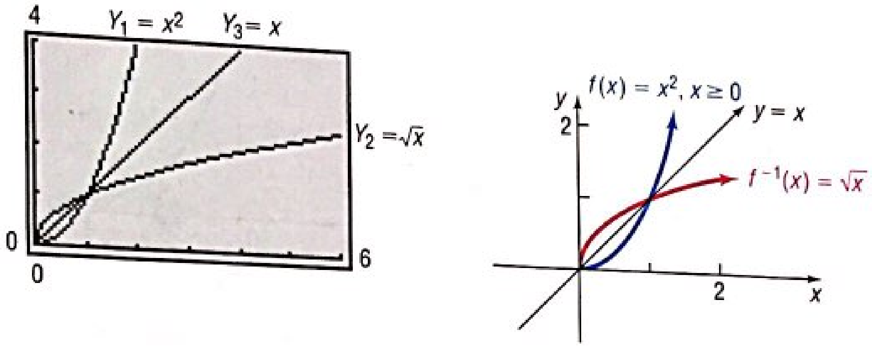
So $f^{-1}(x) = \sqrt{x}$.

STEP 3: Check: $f^{-1}(f(x)) = f^{-1}(x^2) = \sqrt{x^2} = |x| = x$, since $x \geq 0$

$$f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x.$$

Figure 11 illustrates the graphs of $Y_1 = f(x) = x^2$, $x \geq 0$, and $Y_2 = f^{-1}(x) = \sqrt{x}$.

Figure 11



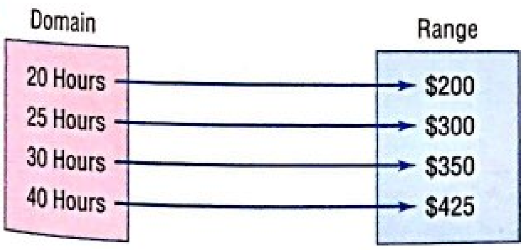
SUMMARY

1. If a function f is one-to-one, then it has an inverse function f^{-1}.
2. Domain f = Range f^{-1}; Range f = Domain f^{-1}.
3. To verify that f^{-1} is the inverse of f, show that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.
4. The graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

5.1 EXERCISES

In Problems 1–8, (a) find the inverse and (b) determine whether the inverse represents a function.

1.



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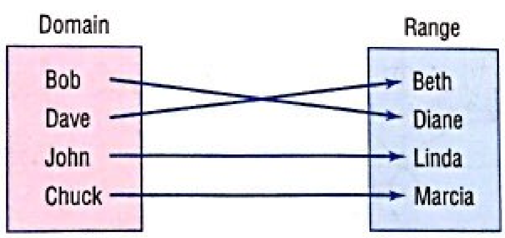

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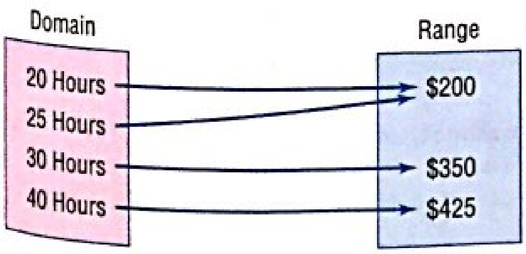

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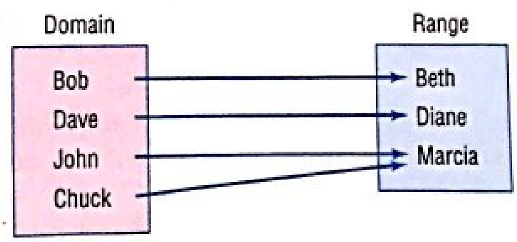


5. $\{(2, 6), (-3, 6), (4, 9), (1, 10)\}$

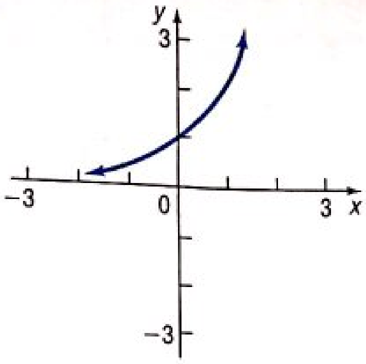
6. $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$

7. $\{(0, 0), (1, 1), (2, 16), (3, 81)\}$

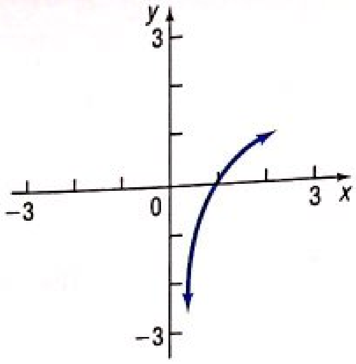
8. $\{(1, 2), (2, 8), (3, 18), (4, 32)\}$

In Problems 9–14, the graph of a function f is given. Use the horizontal line test to determine whether f is one-to-one.

9.



10.



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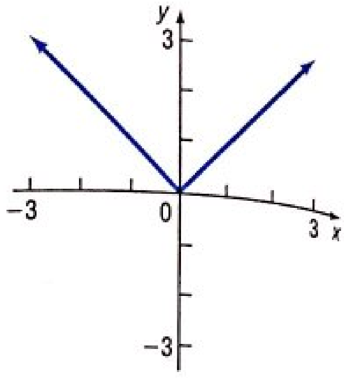

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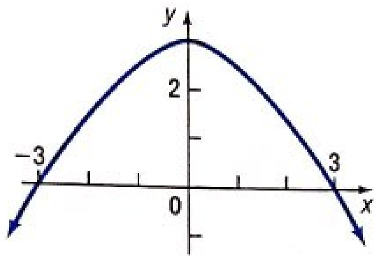
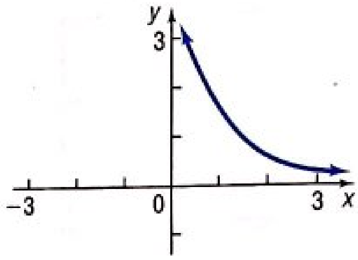
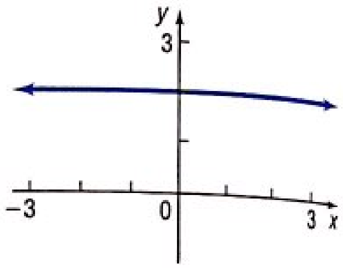


13.

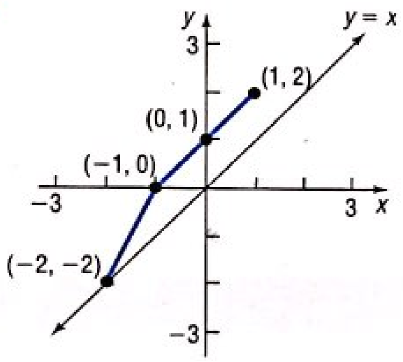


14.

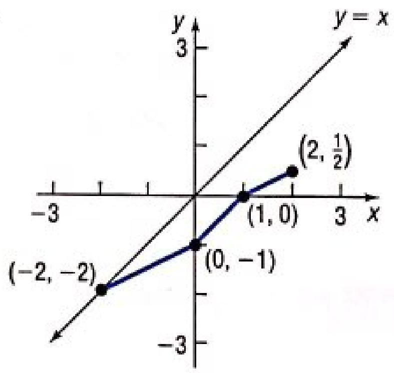


In Problems 15–20, the graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of $y = x$ is also given.

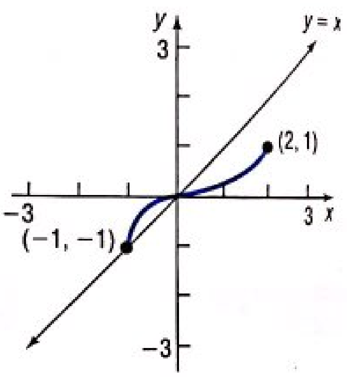
15.



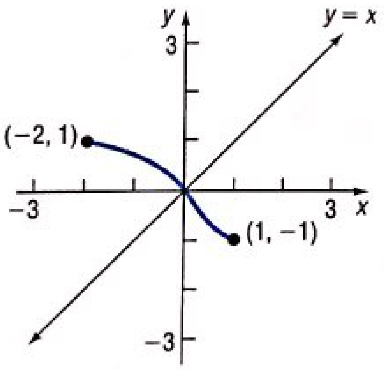
16.



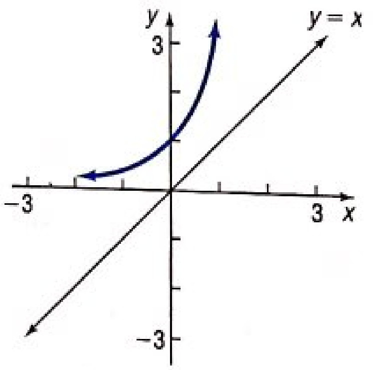
17.



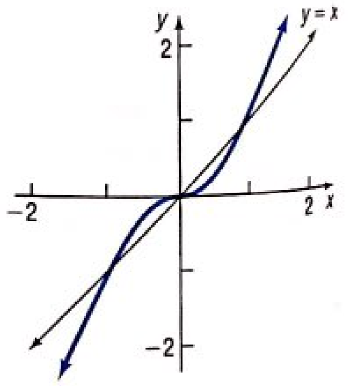
18.



19.



20.



In Problems 21–30, verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Using a graphing utility, simultaneously graph f, g, and $y = x$ on the same square screen.

21. $f(x) = 3x + 4;\quad g(x) = \frac{1}{3}(x - 4)$

22. $f(x) = 3 - 2x;\quad g(x) = -\frac{1}{2}(x - 3)$

23. $f(x) = 4x - 8;\quad g(x) = \frac{x}{4} + 2$

24. $f(x) = 2x + 6;\quad g(x) = \frac{1}{2}x - 3$

25. $f(x) = x^3 - 8;\quad g(x) = \sqrt[3]{x + 8}$

26. $f(x) = (x - 2)^2,\ x \geq 2;\quad g(x) = \sqrt{x} + 2,\ x \geq 0$

27. $f(x) = \frac{1}{x}$; $g(x) = \frac{1}{x}$

28. $f(x) = x$; $g(x) = x$

29. $f(x) = \frac{2x+3}{x+4}$; $g(x) = \frac{4x-3}{2-x}$

30. $f(x) = \frac{x-5}{2x+3}$; $g(x) = \frac{3x+5}{1-2x}$

In Problems 31–42, the function f is one-to-one. Find its inverse and check your answer. State the domain and range of f and f^{-1}. By hand, graph f, f^{-1}, and $y = x$ on the same coordinate axes. Check your results using a graphing utility.

31. $f(x) = 3x$

32. $f(x) = -4x$

33. $f(x) = 4x + 2$

34. $f(x) = 1 - 3x$

35. $f(x) = x^3 - 1$

36. $f(x) = x^3 + 1$

37. $f(x) = x^2 + 4, \quad x \ge 0$

38. $f(x) = x^2 + 9, \quad x \ge 0$

39. $f(x) = \frac{4}{x}$

40. $f(x) = -\frac{3}{x}$

41. $f(x) = \frac{1}{x-2}$

42. $f(x) = \frac{4}{x+2}$

In Problems 43–54, the function f is one-to-one. Find its inverse and check your answer. State the domain and range of f and f^{-1}. Using a graphing utility, simultaneously graph f, f^{-1}, and $y = x$ on the same square screen.

43. $f(x) = \frac{2}{3+x}$

44. $f(x) = \frac{4}{2-x}$

45. $f(x) = (x+2)^2, \quad x \ge -2$

46. $f(x) = (x-1)^2, \quad x \ge 1$

47. $f(x) = \frac{2x}{x-1}$

48. $f(x) = \frac{3x+1}{x}$

49. $f(x) = \frac{3x+4}{2x-3}$

50. $f(x) = \frac{2x-3}{x+4}$

51. $f(x) = \frac{2x+3}{x+2}$

52. $f(x) = \frac{-3x-4}{x-2}$

53. $f(x) = 2\sqrt[3]{x}$

54. $f(x) = \frac{4}{\sqrt{x}}$

55. Find the inverse of the linear function
$f(x) = mx + b, \quad m \ne 0$

56. Find the inverse of the function
$f(x) = \sqrt{r^2 - x^2}, \quad 0 \le x \le r$

57. Can an even function be one-to-one? Explain.

58. Is every odd function one-to-one? Explain.

59. A function f has an inverse function. If the graph of f lies in quadrant I, in which quadrant does the graph of f^{-1} lie?

60. A function f has an inverse function. If the graph of f lies in quadrant II, in which quadrant does the graph of f^{-1} lie?

61. The function $f(x) = |x|$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

62. The function $f(x) = x^4$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

63. **Temperature Conversion** To convert from x degrees Celsius to y degrees Fahrenheit, we use the formula $y = f(x) = \frac{9}{5}x + 32$. To convert from x degrees Fahrenheit to y degrees Celsius, we use the formula $y = g(x) = \frac{5}{9}(x - 32)$. Show that f and g are inverse functions.

64. **Demand for Corn** The demand for corn obeys the equation $p(x) = 300 - 50x$, where p is the price per bushel (in dollars) and x is the number of bushels produced, in millions. Express the production amount x as a function of the price p.

65. **Period of a Pendulum** The period T (in seconds) of a simple pendulum is a function of its length l (in feet), given by $T(l) = 2\pi\sqrt{l/g}$, where $g \approx 32.2$ feet per second per second is the acceleration of gravity. Express the length l as a function of the period T.

66. Give an example of a function whose domain is the set of real numbers and that is neither increasing nor decreasing on its domain, but is one-to-one.
[**Hint:** Use a piecewise-defined function.]

67. Given

$$f(x) = \frac{ax+b}{cx+d}$$

find $f^{-1}(x)$. If $c \ne 0$, under what conditions on a, b, c, and d is $f = f^{-1}$?

68. We said earlier that finding the range of a function f is not easy. However, if f is one-to-one, we can find its range by finding the domain of the inverse function f^{-1}. Use this technique to find the range of each of the following one-to-one functions:

(a) $f(x) = \frac{2x+5}{x-3}$ (b) $g(x) = 4 - \frac{2}{x}$

(c) $F(x) = \frac{3}{4-x}$

69. If the graph of a function and its inverse intersect, where must this necessarily occur? Can they intersect anywhere else? Must they intersect?

70. Can a one-to-one function and its inverse be equal? What must be true about the graph of f for this to happen? Give some examples to support your conclusion.

71. Draw the graph of a one-to-one function that contains the points $(-2, -3)$, $(0, 0)$, and $(1, 5)$. Now draw the graph of its inverse. Compare your graph to those of other students. Discuss any similarities. What differences do you see?