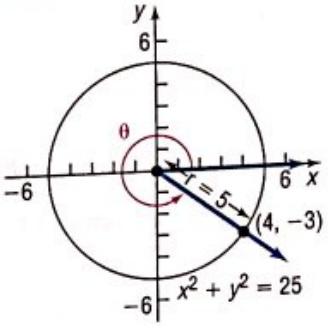
Solution Figure 34 illustrates the situation for θ a positive angle. For th
$(x, y) = (4, -3)$, we have $x = 4$ and $y = -3$. Since $r = \sqrt{x}$
$\sqrt{16 + 9} = 5$, the point $(4, -3)$ is also on the circle $x^2 + y^2 = 25$.

Figure 34



$$\sin\theta = \frac{y}{r} = -\frac{3}{5} \qquad \cos\theta = \frac{x}{r} = \frac{4}{5} \qquad \tan\theta = \frac{y}{x} = -\frac{3}{4}$$

$$\csc\theta = \frac{r}{y} = -\frac{5}{3} \qquad \sec\theta = \frac{r}{x} = \frac{5}{4} \qquad \cot\theta = \frac{x}{y} = -\frac{4}{3}$$

NOW WORK PROBLEM 89.

HISTORICAL FEATURE

The name *sine* for the sine function is due to a medieval confusion. The name comes from the Sanskrit word *jiva* (meaning chord), first used in India by Aryabhata the Elder (AD 510). He really meant half-chord, but abbreviated it. This was brought into Arabic as *ji ba*, which was meaningless. Because the proper Arabic word *jaib* would be written the same way (short vowels are not written out in Arabic), *ji ba* was pronounced as *jaib*, which meant bosom or hollow, and *jaib* remains as the Arabic word for sine to this day. Scholars translating the Arabic works into Latin found that the word *sinus* also meant bosom or hollow, and from *sinus* we get the word *sine*.

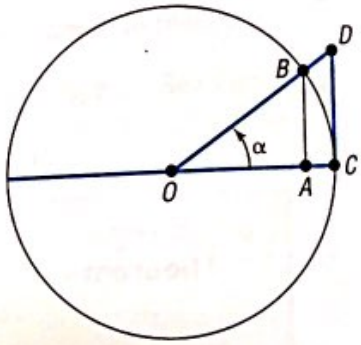
The name *tangent*, due to Thomas Finck (1583), can be understood by looking at Figure 35. The line segment $\overline{DC}$ is tangent to the circle at C. If $d(O, B) = d(O, C) = 1$, then the length of the line segment $\overline{DC}$ is

$$d(D, C) = \frac{d(D, C)}{1} = \frac{d(D, C)}{d(O, C)} = \tan\alpha$$

The old name for the tangent is *umbra versa* (meaning turned shadow), referring to the use of the tangent in solving height problems with shadows.

The names of the remaining functions came about as follows. If α and β are complementary angles, then $\cos\alpha = \sin\beta$. Because β is the complement of α, it was natural to write the cosine of α as *sin co* α. Probably for reasons involving ease of pronunciation, the *co* migrated to the front, and then cosine received a three-letter abbreviation to match sin, sec, and tan. The two other cofunctions were similarly treated, except that the long forms *cotan* and *cosec* survive to this day in some countries.

Figure 35



6.2 EXERCISES

In Problems 1–10, t is a real number and $P = (x, y)$ is the point on the unit circle that corresponds to t. Find the exact value of the six trigonometric functions of t.

1. $\left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$
2. $\left(\frac{3}{8}, \frac{\sqrt{55}}{8}\right)$
3. $\left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$
4. $\left(-\frac{1}{5}, \frac{2\sqrt{6}}{5}\right)$
5. $\left(\frac{-\sqrt{35}}{6}, -\frac{1}{6}\right)$
6. $\left(\frac{-\sqrt{39}}{8}, \frac{5}{8}\right)$
7. $\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$
8. $\left(\frac{-\sqrt{5}}{3}, -\frac{2}{3}\right)$
9. $\left(-\frac{3\sqrt{5}}{7}, \frac{2}{7}\right)$
10. $\left(-\frac{3\sqrt{11}}{10}, -\frac{1}{10}\right)$

In Problems 11–30, find the exact value of each expression. Do not use a calculator.

11. $\sin 45° + \cos 60°$
12. $\sin 30° - \cos 45°$
13. $\sin 90° + \tan 45°$
14. $\cos 180° - \sin 180°$
15. $\sin 45° \cos 45°$
16. $\tan 45° \cos 30°$

17. $\csc 45° \tan 60°$
18. $\sec 30° \cot 45°$
19. $4 \sin 90° - 3 \tan 180°$
20. $5 \cos 90° - 8 \sin 270°$
21. $2 \sin \frac{\pi}{3} - 3 \tan \frac{\pi}{6}$
22. $2 \sin \frac{\pi}{4} + 3 \tan \frac{\pi}{4}$
23. $\sin \frac{\pi}{4} - \cos \frac{\pi}{4}$
24. $\tan \frac{\pi}{3} + \cos \frac{\pi}{3}$
25. $2 \sec \frac{\pi}{4} + 4 \cot \frac{\pi}{3}$
26. $3 \csc \frac{\pi}{3} + \cot \frac{\pi}{4}$
27. $\tan \pi - \cos 0$
28. $\sin \frac{3\pi}{2} + \tan \pi$
29. $\csc \frac{\pi}{2} + \cot \frac{\pi}{2}$
30. $\sec \pi - \csc \frac{\pi}{2}$

In Problems 31–52, find the exact value of the six trigonometric functions of the given angle. If any are not defined, say "not defined." Do not use a calculator.

31. $\frac{2\pi}{3}$
32. $\frac{5\pi}{6}$
33. $210°$
34. $240°$
35. $\frac{5\pi}{3}$
36. $\frac{11\pi}{6}$
37. $7\pi/3$
38. $13\pi/6$
39. $405°$
40. $390°$
41. $-\pi/6$
42. $-\pi/3$
43. $-45°$
44. $-60°$
45. $5\pi/2$
46. 3π
47. $-180°$
48. $-270°$
49. $-\pi/2$
50. -5π
51. $480°$
52. $-150°$

In Problems 53–70, use a calculator to find the approximate value of each expression rounded to two decimal places.

53. $\sin 28°$
54. $\cos 14°$
55. $\tan 21°$
56. $\sin 15°$
57. $\sec 41°$
58. $\csc 55°$
59. $\cot 70°$
60. $\tan 80°$
61. $\sin \frac{\pi}{10}$
62. $\cos \frac{\pi}{8}$
63. $\tan \frac{5\pi}{12}$
64. $\sin \frac{3\pi}{10}$
65. $\sec \frac{\pi}{12}$
66. $\csc \frac{5\pi}{13}$
67. $\sin 1$
68. $\tan 1$
69. $\sin 1°$
70. $\tan 1°$

In Problems 71–82, $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$. Find the exact value of each function below if $\theta = 60°$. Do not use a calculator.

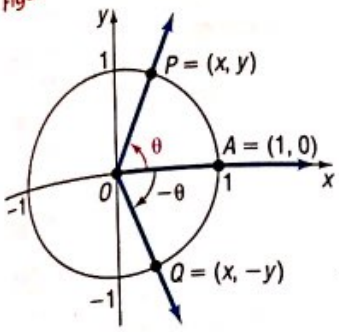
71. $f(\theta)$
72. $g(\theta)$
73. $f\left(\frac{\theta}{2}\right)$
74. $g\left(\frac{\theta}{2}\right)$
75. $[f(\theta)]^2$
76. $[g(\theta)]^2$
77. $f(2\theta)$
78. $g(2\theta)$
79. $2f(\theta)$
80. $2g(\theta)$
81. $f(-\theta)$
82. $g(-\theta)$

83. Use a calculator in radian mode to complete the following table.
What can you conclude about the ratio $(\sin\theta)/\theta$ as θ approaches 0?

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$								
$\frac{\sin\theta}{\theta}$								

84. Use a calculator in radian mode to complete the following table.
What can you conclude about the ratio $(\cos\theta - 1)/\theta$ as θ approaches 0?

Figure 43



to the angle $-\theta$ will have coordinates $(x, -y)$. Using the definition for the trigonometric functions, we have

$$\sin\theta = y \qquad \cos\theta = x \qquad \sin(-\theta) = -y \qquad \cos(-\theta) = x$$

so

$$\sin(-\theta) = -y = -\sin\theta \qquad \cos(-\theta) = x = \cos\theta$$

Now, using these results and some of the fundamental identities, we have

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta \qquad \cot(-\theta) = \frac{1}{\tan(-\theta)} = \frac{1}{-\tan\theta} = -\cot\theta$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \sec\theta \qquad \csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\csc\theta$$

SEEING THE CONCEPT To see that the cosine function is even, create a TABLE with $Y_1 = \cos x$, $Y_2 = \cos(-x)$, and TblStart = 0, and ΔTbl = $\pi/6$. To see that the sine function is odd, repeat this procedure with $Y_1 = -\sin x$ and $Y_2 = \sin(-x)$.

EXAMPLE 5 Finding Exact Values Using Even–Odd Properties

Find the exact value of:

(a) $\sin(-45°)$ (b) $\cos(-\pi)$ (c) $\cot(-3\pi/2)$ (d) $\tan(-37\pi/4)$

Solution

(a) $\sin(-45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$ (Odd function)

(b) $\cos(-\pi) = \cos\pi = -1$ (Even function)

(c) $\cot\left(-\frac{3\pi}{2}\right) = -\cot\frac{3\pi}{2} = 0$ (Odd function)

(d) $\tan\left(-\frac{37\pi}{4}\right) = -\tan\frac{37\pi}{4} = -\tan\left(\frac{\pi}{4} + 9\pi\right) = -\tan\frac{\pi}{4} = -1$ (Odd function; Period is π)

NOW WORK PROBLEM 49.

6.3 EXERCISES

In Problems 1–16, use the fact that the trigonometric functions are periodic to find the exact value of each expression. Do not use a calculator.

1. $\sin 405°$
2. $\cos 420°$
3. $\tan 405°$
4. $\sin 390°$
5. $\csc 450°$
6. $\sec 540°$
7. $\cot 390°$
8. $\sec 420°$
9. $\cos\frac{33\pi}{4}$
10. $\sin\frac{9\pi}{4}$
11. $\tan(21\pi)$
12. $\csc\frac{9\pi}{2}$
13. $\sec\frac{17\pi}{4}$
14. $\cot\frac{17\pi}{4}$
15. $\tan\frac{19\pi}{6}$
16. $\sec\frac{25\pi}{6}$

In Problems 17–24, name the quadrant in which the angle θ lies.

17. $\sin\theta > 0, \quad \cos\theta < 0$
18. $\sin\theta < 0, \quad \cos\theta > 0$
19. $\sin\theta < 0, \quad \tan\theta < 0$
20. $\cos\theta > 0, \quad \tan\theta > 0$
21. $\cos\theta > 0, \quad \tan\theta < 0$
22. $\cos\theta < 0, \quad \tan\theta > 0$
23. $\sec\theta < 0, \quad \sin\theta > 0$
24. $\csc\theta > 0, \quad \cos\theta < 0$

In Problems 25–32, $\sin\theta$ and $\cos\theta$ are given. Find the exact value of each of the four remaining trigonometric functions. In Problems 31 and 32, round your answer to four decimal places.

25. $\sin\theta = 2\sqrt{5}/5, \quad \cos\theta = \sqrt{5}/5$
26. $\sin\theta = -\sqrt{5}/5, \quad \cos\theta = -2\sqrt{5}/5$
27. $\sin\theta = \frac{1}{2}, \quad \cos\theta = \sqrt{3}/2$
28. $\sin\theta = \sqrt{3}/2, \quad \cos\theta = \frac{1}{2}$
29. $\sin\theta = -\frac{1}{3}, \quad \cos\theta = 2\sqrt{2}/3$
30. $\sin\theta = 2\sqrt{2}/3, \quad \cos\theta = -\frac{1}{3}$
31. $\sin\theta = 0.2588, \quad \cos\theta = 0.9659$
32. $\sin\theta = 0.6428, \quad \cos\theta = 0.7660$

In Problems 33–48, find the exact value of each of the remaining trigonometric functions of θ.

33. $\sin\theta = \frac{12}{13}$, θ in quadrant II
34. $\cos\theta = \frac{3}{5}$, θ in quadrant IV
35. $\cos\theta = -\frac{4}{5}$, θ in quadrant III
36. $\sin\theta = -\frac{5}{13}$, θ in quadrant III
37. $\sin\theta = \frac{5}{13}$, $90° < \theta < 180°$
38. $\cos\theta = \frac{4}{5}$, $270° < \theta < 360°$
39. $\cos\theta = -\frac{1}{3}$, $\frac{\pi}{2} < \theta < \pi$
40. $\sin\theta = -\frac{2}{3}$, $\pi < \theta < 3\pi/2$
41. $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$
42. $\cos\theta = -\frac{1}{4}$, $\tan\theta > 0$
43. $\sec\theta = 2$, $\sin\theta < 0$
44. $\csc\theta = 3$, $\cot\theta < 0$
45. $\tan\theta = \frac{3}{4}$, $\sin\theta < 0$
46. $\cot\theta = \frac{4}{3}$, $\cos\theta < 0$
47. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$
48. $\sec\theta = -2$, $\tan\theta > 0$

In Problems 49–66, use the even–odd properties to find the exact value of each expression. Do not use a calculator.

49. $\sin(-60°)$
50. $\cos(-30°)$
51. $\tan(-30°)$
52. $\sin(-135°)$
53. $\sec(-60°)$
54. $\csc(-30°)$
55. $\sin(-90°)$
56. $\cos(-270°)$
57. $\tan\left(-\frac{\pi}{4}\right)$
58. $\sin(-\pi)$
59. $\cos\left(-\frac{\pi}{4}\right)$
60. $\sin\left(-\frac{\pi}{3}\right)$
61. $\tan(-\pi)$
62. $\sin\left(-\frac{3\pi}{2}\right)$
63. $\csc\left(-\frac{\pi}{4}\right)$
64. $\sec(-\pi)$
65. $\sec\left(-\frac{\pi}{6}\right)$
66. $\csc\left(-\frac{\pi}{3}\right)$

In Problems 67–78, find the exact value of each expression. Do not use a calculator.

67. $\sin(-\pi) + \cos(5\pi)$
68. $\tan\left(-\frac{5\pi}{6}\right) - \cot\frac{7\pi}{2}$
69. $\sec(-\pi) + \csc\left(-\frac{\pi}{2}\right)$
70. $\tan(-6\pi) + \cos\frac{9\pi}{4}$
71. $\sin\left(-\frac{9\pi}{4}\right) - \tan\left(-\frac{9\pi}{4}\right)$
72. $\cos\left(-\frac{17\pi}{4}\right) - \sin\left(-\frac{3\pi}{2}\right)$
73. $\sin^2 40° + \cos^2 40°$
74. $\sec^2 18° - \tan^2 18°$
75. $\sin 80° \csc 80°$
76. $\tan 10° \cot 10°$
77. $\tan 40° - \frac{\sin 40°}{\cos 40°}$
78. $\cot 20° - \frac{\cos 20°}{\sin 20°}$

79. If $\sin\theta = 0.3$, find the value of: $\sin\theta + \sin(\theta + 2\pi) + \sin(\theta + 4\pi)$.
80. If $\cos\theta = 0.2$, find the value of: $\cos\theta + \cos(\theta + 2\pi) + \cos(\theta + 4\pi)$.
81. If $\tan\theta = 3$, find the value of: $\tan\theta + \tan(\theta + \pi) + \tan(\theta + 2\pi)$.
82. If $\cot\theta = -2$, find the value of: $\cot\theta + \cot(\theta - \pi) + \cot(\theta - 2\pi)$.
83. Find the exact value of $\sin 1° + \sin 2° + \sin 3° + \cdots + \sin 358° + \sin 359°$.
84. Find the exact value of $\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 358° + \cos 359°$.
85. What is the domain of the sine function?
86. What is the domain of the cosine function?
87. For what numbers θ is $f(\theta) = \tan\theta$ not defined?
88. For what numbers θ is $f(\theta) = \cot\theta$ not defined?
89. For what numbers θ is $f(\theta) = \sec\theta$ not defined?
90. For what numbers θ is $f(\theta) = \csc\theta$ not defined?
91. What is the range of the sine function?
92. What is the range of the cosine function?
93. What is the range of the tangent function?
94. What is the range of the cotangent function?
95. What is the range of the secant function?
96. What is the range of the cosecant function?