$$4r^2 \cos\theta \sin\theta = 9$$

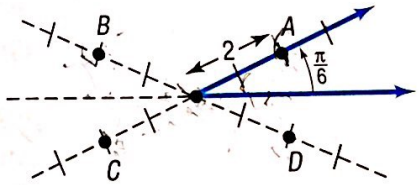
$$2r^2 \sin(2\theta) = 9 \quad \text{Double-angle formula}$$

9.1 EXERCISES

In Problems 1–8, match each point in polar coordinates with either A, B, C, or D on the graph.

1. $\left(2, \frac{-11\pi}{6}\right)$ 2. $\left(-2, \frac{-\pi}{6}\right)$ 3. $\left(-2, \frac{\pi}{6}\right)$ 4. $\left(2, \frac{7\pi}{6}\right)$

5. $\left(2, \frac{5\pi}{6}\right)$ 6. $\left(-2, \frac{5\pi}{6}\right)$ 7. $\left(-2, \frac{7\pi}{6}\right)$ 8. $\left(2, \frac{11\pi}{6}\right)$



In Problems 9–20, plot each point given in polar coordinates.

9. $(3, 90°)$ 10. $(4, 270°)$ 11. $(-2, 0)$ 12. $(-3, \pi)$

13. $(6, \pi/6)$ 14. $(5, 5\pi/3)$ 15. $(-2, 135°)$ 16. $(-3, 120°)$

17. $(-1, -\pi/3)$ 18. $(-3, -3\pi/4)$ 19. $(-2, -\pi)$ 20. $(-3, -\pi/2)$

In Problems 21–28, plot each point given in polar coordinates, and find other polar coordinates (r, θ) of the point for which:

(a) $r > 0, \quad -2\pi \le \theta < 0$ *(b)* $r < 0, \quad 0 \le \theta < 2\pi$ *(c)* $r > 0, \quad 2\pi \le \theta < 4\pi$

21. $(5, 2\pi/3)$ 22. $(4, 3\pi/4)$ 23. $(-2, 3\pi)$ 24. $(-3, 4\pi)$

25. $(1, \pi/2)$ 26. $(2, \pi)$ 27. $(-3, -\pi/4)$ 28. $(-2, -2\pi/3)$

In Problems 29–44, polar coordinates of a point are given. Find the rectangular coordinates of each point.

29. $(3, \pi/2)$ 30. $(4, 3\pi/2)$ 31. $(-2, 0)$ 32. $(-3, \pi)$

33. $(6, 150°)$ 34. $(5, 300°)$ 35. $(-2, 3\pi/4)$ 36. $(-3, 2\pi/3)$

37. $(-1, -\pi/3)$ 38. $(-3, -3\pi/4)$ 39. $(-2, -180°)$ 40. $(-3, -90°)$

41. $(7.5, 110°)$ 42. $(-3.1, 182°)$ 43. $(6.3, 3.8)$ 44. $(8.1, 5.2)$

In Problems 45–56, the rectangular coordinates of a point are given. Find polar coordinates for each point.

45. $(3, 0)$ 46. $(0, 2)$ 47. $(-1, 0)$ 48. $(0, -2)$

49. $(1, -1)$ 50. $(-3, 3)$ 51. $(\sqrt{3}, 1)$ 52. $(-2, -2\sqrt{3})$

53. $(1.3, -2.1)$ 54. $(-0.8, -2.1)$ 55. $(8.3, 4.2)$ 56. $(-2.3, 0.2)$

In Problems 57–64, the letters x and y represent rectangular coordinates. Write each equation using polar coordinates (r, θ).

57. $2x^2 + 2y^2 = 3$ **58.** $x^2 + y^2 = x$ **59.** $x^2 = 4y$ **60.** $y^2 = 2x$

61. $2xy = 1$ **62.** $4x^2y = 1$ **63.** $x = 4$ **64.** $y = -3$

In Problems 65–72, the letters r and θ represent polar coordinates. Write each equation using rectangular coordinates (x, y).

65. $r = \cos\theta$ **66.** $r = \sin\theta + 1$ **67.** $r^2 = \cos\theta$ **68.** $r = \sin\theta - \cos\theta$

69. $r = 2$ **70.** $r = 4$ **71.** $r = \dfrac{4}{1 - \cos\theta}$ **72.** $r = \dfrac{3}{3 - \cos\theta}$

73. Show that the formula for the distance d between two points $P_1 = (r_1, \theta_1)$ and $P_2 = (r_2, \theta_2)$ is

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$

PREPARING FOR THIS SECTION

Before getting started, review the following:

- ✓ Graphs of Equations (Section 1.2)
- ✓ Even/Odd Properties of Trigonometric Functions (pp. 404–405)
- ✓ Circles (Section 1.8)
- ✓ Difference Formulas (pp. 452 and 455)

9.2 POLAR EQUATIONS AND GRAPHS

OBJECTIVES

1. Graph and Identify Polar Equations by Converting to Rectangular Equations
2. Graph Polar Equations Using a Graphing Utility
3. Test Polar Equations for Symmetry
4. Graph Polar Equations by Plotting Points

Just as a rectangular grid may be used to plot points given by rectangular coordinates, as in Figure 20(a), we can use a grid consisting of concentric circles (with centers at the pole) and rays (with vertex