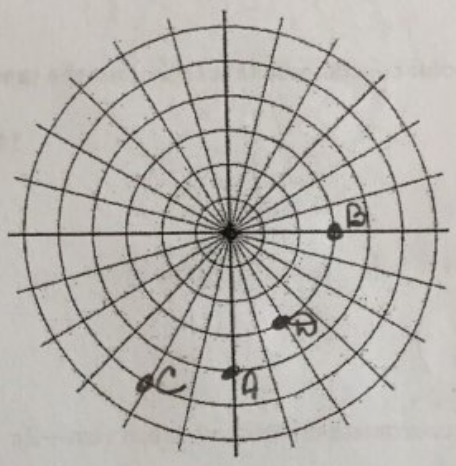
Name: Key

Polar Review

Plot each polar coordinate on the graph.

1. $A(4, 270°)$
2. $B(-3, \pi)$
3. $C\left(5, \frac{4\pi}{3}\right)$
4. $D(-3, 120°)$



Convert each of the following rectangular coordinates to polar coordinates. Answers for θ should be in radians, exact answers if possible. If you need to have decimals, please round to the nearest thousandth.

5. $(-3, 3)$ $(3\sqrt{2}, 135°)$

$r = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}(-1)$

6. $(-2, -2\sqrt{3})$ $(4, 4\pi/3)$

$r = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) = \sqrt{3}$

7. $(-0.8, -2.1)$ $(2.247, 4.348)$

$r = \sqrt{(-.8)^2 + (-2.1)^2}$

$\theta = \tan^{-1}\left(\frac{-2.1}{-.8}\right)$

8. $(-2.3, 0.2)$ $(2.309, 3.055)$

$r = \sqrt{(-2.3)^2 + 0.2^2}$

$\theta = \tan^{-1}\left(\frac{.2}{-2.3}\right)$

$\theta = -.087 + \pi$

QII

Convert each of the following polar coordinates to rectangular coordinates. Decimals should be rounded to the nearest thousandth.

9. $(5, 300°)$ $\left(\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$

$x = 5\cos 300$

$y = 5\sin 300$

10. $\left(4, \frac{3\pi}{2}\right)$ $(0, -4)$

$x = 4\cos 3\pi/2$

$y = 4\sin 3\pi/2$

11. $(-3.1, 182°)$ $(3.098, 0.108)$

$x = -3.1 \cos 182°$

$y = -3.1 \sin 182°$

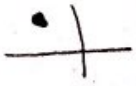
12. $(8.1, 5.2)$ $(3.795, -7.156)$

$x = 8.1 \cos 5.2$

$y = 8.1 \sin 5.2$

Find the other 3 polar coordinates that represent the same point for $-2\pi \le \theta \le 2\pi$.

13. $\left(4, \frac{3\pi}{4}\right)$



$(-4, 7\pi/4)$

$(-4, -\pi/4)$

$(4, 7\pi/4)$

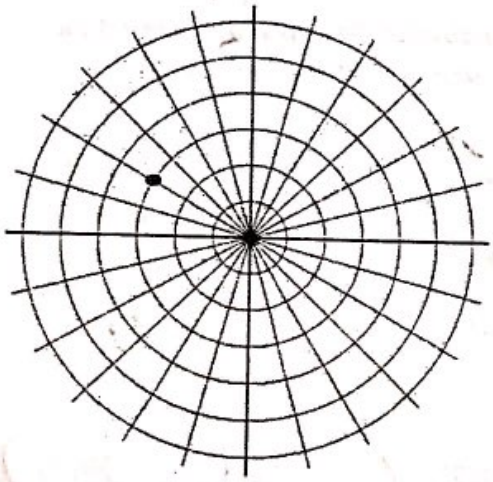
14. $\left(-2, -\frac{2\pi}{3}\right)$

$(2, \pi/3)$

$(2, -5\pi/3)$

$(-2, 4\pi/3)$

15. Find all polar coordinates for the point such that $-2\pi \le \theta \le 2\pi$.



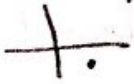
$(2, 5\pi/6)$ $(2, -7\pi/6)$

$(-2, 11\pi/6)$

$(-2, -\pi/6)$

Find the other 3 polar coordinates that represent the same point for $-360° \le \theta \le 360°$.

16. $(3, -20°)$

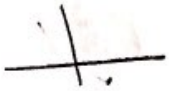


$(3, 340°)$

$(-3, -200°)$

$(-3, 160°)$

17. $(-4, 103°)$



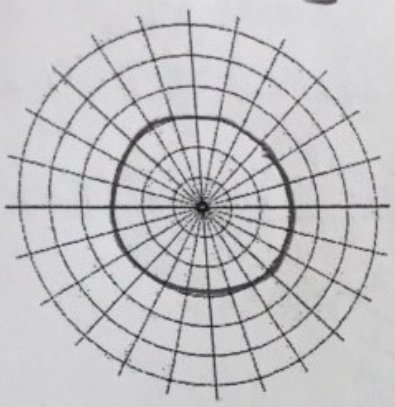
$(-4, -257°)$

$(4, 283°)$

$(4, -77°)$

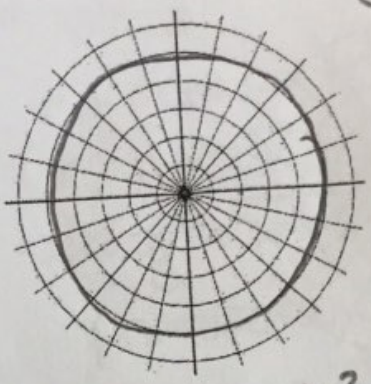
Graph each polar equation. Then convert the equation to rectangular form.

18. $r^2 = 3^2$ $x^2+y^2=9$



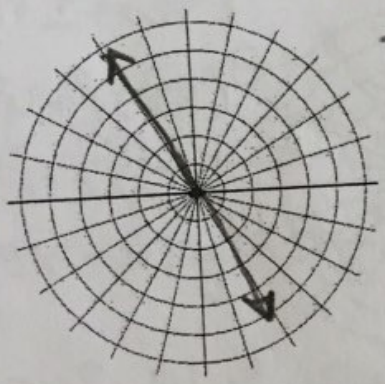
rectangular equation: $x^2+y^2=9$

19. $r^2 = 5^2$ $x^2+y^2=25$



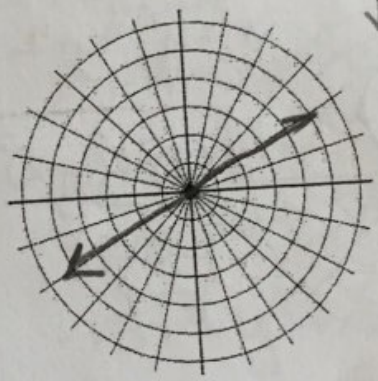
rectangular equation: $x^2+y^2=25$

18. $\theta = \frac{2\pi}{3}$ $\tan\theta = y/x$, $-\sqrt{3} = y/x$



rectangular equation: $y=-x\sqrt{3}$

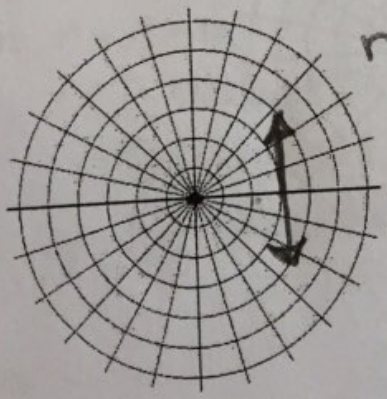
19. $\theta = 30°$ $\tan\theta = y/x$, $\sqrt{3}/3 = y/x$



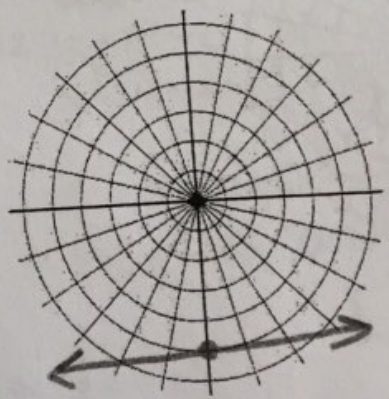
rectangular equation: $y = x\frac{\sqrt{3}}{3}$

Convert the following polar equations into rectangular equations so that you can graph them.

20. $r = 3\sec\theta$ $r = \frac{3}{\cos\theta}$, $r\cos\theta = 3$, $x = 3$



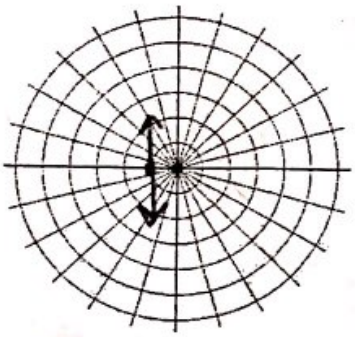
21. $r = -5\csc\theta$ $r = \frac{-5}{\sin\theta}$, $r\sin\theta = -5$, $y = -5$



22. $r = -\frac{1}{\cos\theta}$

$r\cos\theta = -1$

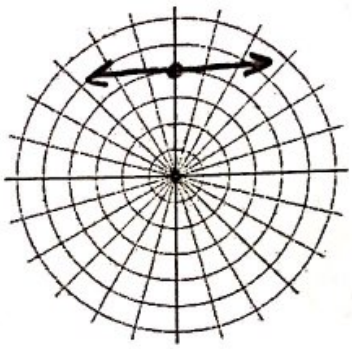
$x = -1$



23. $r = \frac{4}{\sin\theta}$

$r\sin\theta = 4$

$y = 4$



24. $r = 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

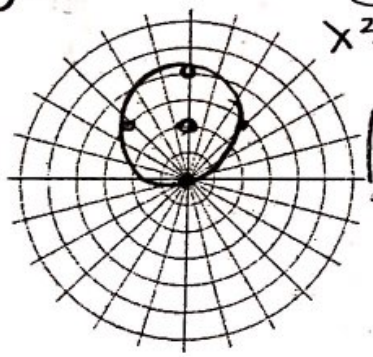
$x^2 + y^2 - 4y + 4 = 0 + 4$

$(x)^2 + (y-2)^2 = 4$

Circle

$C(0,2)$

$r = 2$



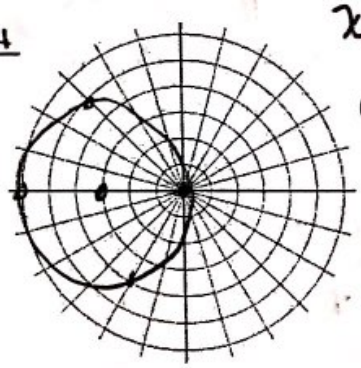
25. $r = -6\cos\theta$

$x^2 + y^2 = -6x$

$x^2 + 6x + 9 + y^2 = 0 + 9$

$(x+3)^2 + y^2 = 3$

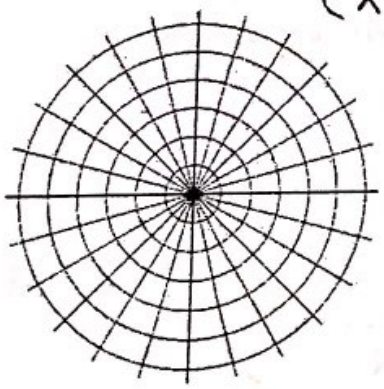
Circle



25. $r = 5\cos\theta$

$(x - \frac{5}{2})^2 + y^2 = 6.25$

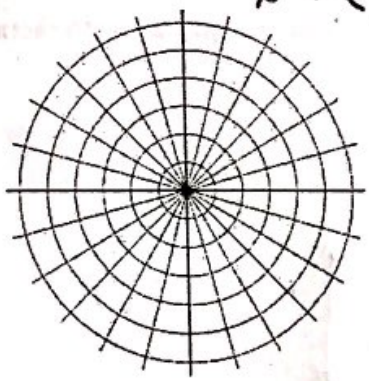
Circle



26. $r = -2\sin\theta$

$x^2 + (y+1)^2 = 1$

circle



conic.

27. $r = 2\sin\theta - 6\cos\theta$

$x^2+y^2 = 2y-6x$

$(x+3)^2+(y-1)^2=10$

Circle

28. $r=\frac{1}{1+\cos\theta}$

$r+r\cos\theta=1$

$(r)^2=(1-x)^2$

$x^2+y^2=x^2-2x+1$

parabola

$x=-\frac{1}{2}y^2+\frac{1}{2}$

29. $r=\frac{3}{5-2\sin\theta}$

$5r-2r\sin\theta=3$ ellipse

$(5r)^2=(2y+3)^2$

$25x^2+25y^2=4y^2+12y+9$

$25x^2+21y^2-12y-45=0$

30. $r=\frac{2}{1+4\cos\theta}$ hyperbola

$r+4r\cos\theta=2$

$r^2=(2-4x)^2$

$x^2+y^2=16x^2-16x+4$

$-15x^2+y^2+16x-4=0$

Convert each of the following rectangular equations to polar form.

31. $x^2+y^2=4x$ $r=4\cos\theta$

$r^2=4r\cos\theta$

$r^2-4r\cos\theta=0$

$r(r-4\cos\theta)=0$

32. $(x-7)^2+(y+2)^2=53$

$x^2-14x+49+y^2+4y+4=53$

$r^2-14\cos\theta+4r\sin\theta=0$

$r(r-14\cos\theta+4\sin\theta)=0$

$r=14\cos\theta-4\sin$

33. $y=-x$

$\frac{y}{x}=-1$

$\tan\theta=-45$

135

$\theta=135°$

or

$\theta=315°$

34. $y=\frac{\sqrt{3}}{3}x$

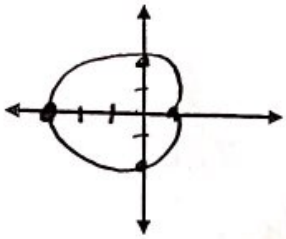
$\frac{y}{x}=\frac{\sqrt{3}}{3}$

$\tan\theta=\pi/6$ $\theta=\pi/6$ or $\theta=7$

Identify and sketch a graph for the following.

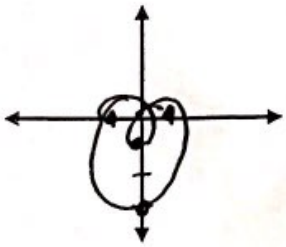
35. $r = 2 - \cos\theta$

identification: lim w/out a loop



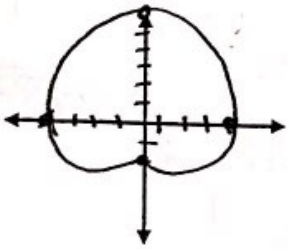
36. $r = 1 - 2\sin\theta$

identification: lim w/ a loop



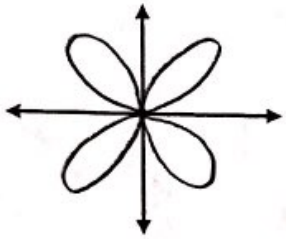
37. $r = 4 + 2\sin\theta$

identification: lim w/out loop



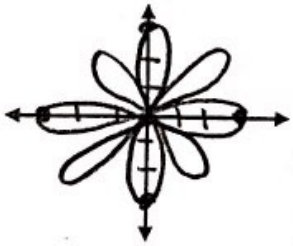
38. $r = 2\sin 2\theta$

identification: rose 4 petals



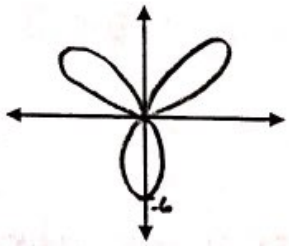
39. $r = 3\cos 4\theta$

identification: rose 8 petals



40. $r = 6\sin 3\theta$ rose 3 petals

identification:



Eliminate the parameter.

24. $x = t - 3, \quad y = 2t + 1, \quad t \geq 0$

$t = x + 3 \quad y = 2(x+3) + 1$

$y = 2x + 7$

25. $x = t + 5, \quad y = \sqrt{t}, \quad t \geq 0$

$t = x - 5 \quad y = \sqrt{x-5}$

26. $x = -2 + t^2, \quad y = 1 + 2t^2, \quad$ for any t

$\pm\sqrt{x+2}$

$y = 1 + 2(\sqrt{x+2})^2 \quad y = 2x + 5$

$y = 1 + 2x + 4$

27. $x = e^t, \quad y = t, \quad$ for any t

$\ln x = t$

$y = \ln x$

28. $x = 3\cos t, \quad y = 3\sin t, \quad 0 \leq t \leq 2\pi$

$\frac{x}{3} = \cos t^2 \quad \frac{x^2}{9} + \frac{y^2}{9} = 1$

$\frac{y}{3} = \sin t^2$

29. $x = 2\sin t - 3, \quad y = 2\cos t + 1, \quad 0 \leq t \leq 2\pi$

$\frac{x+3}{2} = \sin t \quad \frac{(x+3)^2}{4} + \frac{(y-1)^2}{4} = 1$

$\frac{y-1}{2} = \cos t$

30. A golfer at a driving range stands on a platform 2 feet above the ground and hits the ball with an initial velocity of 120 feet/second at an angle of 39° with the horizontal. There is a 32-foot-high fence 400 feet away. Determine the position of the ball as a pair of parametric equations. Will the ball fall short, hit the fence, or go over the wall?

y max 100

$x = 120\cos 39°(t)$

$y = -16t^2 + 120\sin 39° t + 2$

when does it land : time in air : 4.746 sec

time at 32 feet high 4.282 sec @ 32 feet

distance at 4.282 sec

$x = 120\cos(39)(4.282) = 399.33$ ft

Does Not Clear fence

conversion! Skip!

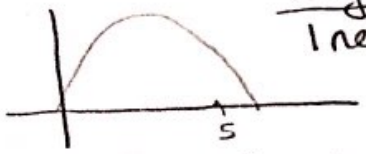
31. Suppose a professional football player kicks a football with an initial velocity of 29 yards per second at an angle of 68° to the horizontal. Suppose a kick returner catches the ball 5 seconds later. Determine the position of the ball as a pair of parametric equations. How far has the ball traveled horizontally and what is its vertical height at the time it is caught?

y-max 20

$x = 87\cos 68° t$

$y = -16t^2 + 87\sin 68t + 0$

$\frac{29\text{yd}}{1\text{sec}} = \frac{3\text{ft}}{1\text{yd}}$



$t = 5$ sec : $87\cos(68)(5) = 162$ ft 3 ft horiz travele

In air 5.041 sec