

Precalculus

Polar Practice

Name \_\_\_\_\_

Date \_\_\_\_\_

Convert each pair of polar coordinates to rectangular coordinates.

1)  $(4, \frac{\pi}{3})$   $x = 4 \cos \frac{\pi}{3} = 4(\frac{1}{2}) = 2$   
 $y = 4 \sin \frac{\pi}{3} = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$   
 Rectangular coordinates:  $(2, 2\sqrt{3})$

2)  $(2, 100^\circ)$   $x = 2 \cos 100^\circ \approx -0.35$   
 $y = 2 \sin 100^\circ \approx 2$   
 Rectangular coordinates:  $(-0.35, 2)$

3)  $(2, \frac{\pi}{6})$   $x = 2 \cos \frac{\pi}{6} = 2(\frac{\sqrt{3}}{2}) = \sqrt{3}$   
 $y = 2 \sin \frac{\pi}{6} = 2(\frac{1}{2}) = 1$   
 Rectangular coordinates:  $(\sqrt{3}, 1)$

4)  $(2, \frac{\pi}{5})$   $x = 2 \cos \frac{\pi}{5} \approx 1.6$   
 $y = 2 \sin \frac{\pi}{5} \approx 1.2$   
 Rectangular coordinates:  $(1.6, 1.2)$

Convert each pair of rectangular coordinates to polar coordinates where  $r > 0$  and  $0 \leq \theta < 2\pi$ .

5)  $(\frac{3\sqrt{3}}{2}, -\frac{3}{2})$   $r = \sqrt{(\frac{3\sqrt{3}}{2})^2 + (-\frac{3}{2})^2} = 3$   
 $\theta = \tan^{-1}(\frac{-3/2}{3\sqrt{3}/2}) = \tan^{-1}(-\frac{1}{\sqrt{3}}) = \frac{11\pi}{6}$   
 Polar coordinates:  $(3, \frac{11\pi}{6})$

6)  $(-\sqrt{2}, -\sqrt{2})$   $r = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = 2$   
 $\theta = \tan^{-1}(\frac{-\sqrt{2}}{-\sqrt{2}}) = \tan^{-1}(1) = \frac{5\pi}{4}$   
 Polar coordinates:  $(2, \frac{5\pi}{4})$

7)  $(-3, 1)$   $r = \sqrt{9+1} = \sqrt{10}$   
 $\theta = \tan^{-1}(\frac{1}{-3}) = \tan^{-1}(-\frac{1}{3})$   
 Polar coordinates:  $(\sqrt{10}, \theta)$

8)  $(5, -4)$   $r = \sqrt{25+16} = \sqrt{41}$   
 $\theta = \tan^{-1}(\frac{-4}{5}) = \tan^{-1}(-\frac{4}{5})$   
 Polar coordinates:  $(\sqrt{41}, \theta)$

Find 3 other ways to write the same point.

9)  $(1, \frac{\pi}{6})$   $(1, -\frac{11\pi}{6})$   
 $(-1, \frac{7\pi}{6})$   
 $(-1, -\frac{5\pi}{6})$

10)  $(3, 80^\circ)$   $(3, -280^\circ)$   
 $(-3, 260^\circ)$   
 $(-3, -100^\circ)$

Change the following equations to rectangular form.

11)  $r = \frac{6}{1 + \sin \theta}$

$r + r \sin \theta = 6$   
 $\sqrt{x^2 + y^2} + \frac{y}{\sqrt{x^2 + y^2}} \sqrt{x^2 + y^2} = 6$   
 $\sqrt{x^2 + y^2} + y = 6$   
 $\sqrt{x^2 + y^2} = 6 - y$   
 $x^2 + y^2 = 36 - 12y + y^2$   
 $x^2 + 12y = 36$   
 Parabola

12)  $r = \frac{6}{1 + 2 \sin \theta}$

$r + 2r \sin \theta = 6$   
 $\sqrt{x^2 + y^2} + 2 \frac{y}{\sqrt{x^2 + y^2}} \sqrt{x^2 + y^2} = 6$   
 $\sqrt{x^2 + y^2} + 2y = 6$   
 $x^2 + y^2 = 36 - 24y + 4y^2$   
 $x^2 - 3y^2 + 24y = 36$   
 hyperbola

$$13) r = \frac{1}{1+4\cos\theta}$$

$$+4r\cos\theta = 1$$

$$x^2 + y^2 + 4x = 1$$

$$x^2 + y^2 = 1 - 8x + 16x^2$$

$$-15x^2 + y^2 = 1 - 8x$$

hyperbola

$$15) \tan\theta = 5$$

$$\frac{y}{x} = 5$$

$$y = 5x$$

linear

$$14) r = 2\cos\theta + 2\sin\theta$$

$$r^2 = 2r\cos\theta + 2r\sin\theta$$

$$x^2 + y^2 = 2x + 2y$$

circle

$$16) r = 4\cos\theta$$

$$r^2 = 4r\cos\theta$$

$$x^2 + y^2 = 4x$$

circle

Convert each equation from rectangular to polar form.

$$17) (x+2)^2 + (y-2)^2 = 8$$

$$r = -4\cos\theta + 4\sin\theta$$

$$18) (x+1)^2 + (y-1)^2 = 2$$

$$r = -2\cos\theta + 2\sin\theta$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 8$$

$$r^2 + 4r\cos\theta - 4r\sin\theta = 0$$

$$19) x^2 + (y+2)^2 = 4$$

$$x^2 + y^2 + 4y + 4 = 4$$

$$r^2 + 4r\sin\theta = 0$$

$$r(r + 4\sin\theta) = 0$$

$$r = 0 \text{ (} r = -4\sin\theta \text{)}$$

$$20) y = \frac{x+1}{x}$$

$$\frac{y}{x} = 1$$

$$\tan\theta = 1$$

$$\theta = \pi/4$$

$$y - x = 0$$

$$r\sin\theta - r\cos\theta = 0$$

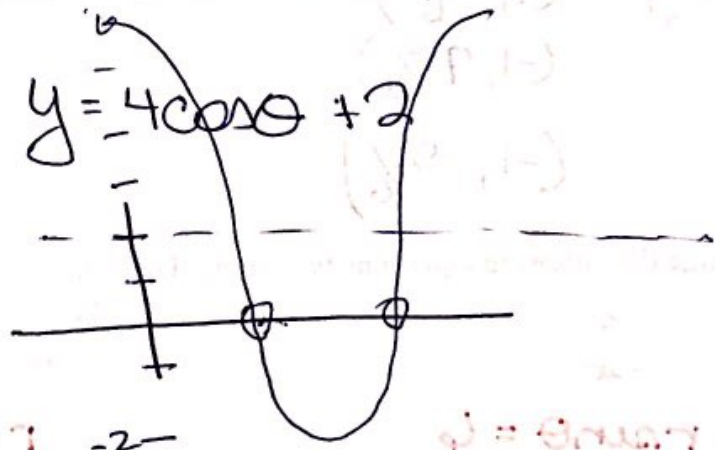
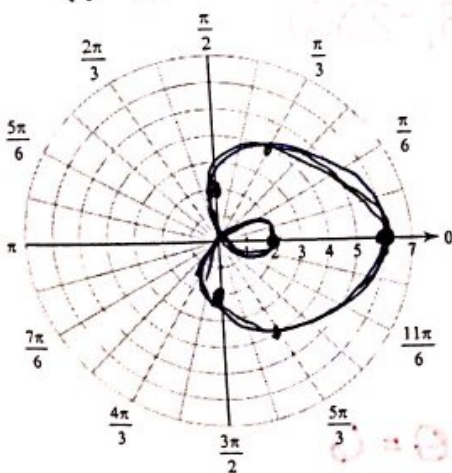
$$r(\sin\theta - \cos\theta) = 0$$

$$r = 0$$

Consider each polar equation. Classify the curve; and sketch the graph.

$$21) r = 2 + 4\cos\theta$$

limaçon w/ inner loop



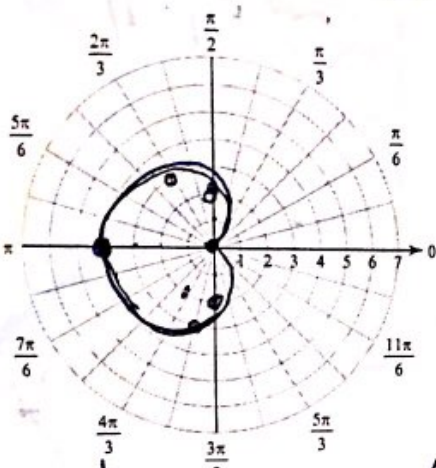
$x = r\cos\theta$   
 $y = r\sin\theta$   
 $x^2 + y^2 = r^2$   
 $r = \sqrt{x^2 + y^2}$   
 $\cos\theta = \frac{x}{r}$   
 $\sin\theta = \frac{y}{r}$

$x = r\cos\theta$   
 $y = r\sin\theta$   
 $x^2 + y^2 = r^2$   
 $r = \sqrt{x^2 + y^2}$   
 $\cos\theta = \frac{x}{r}$   
 $\sin\theta = \frac{y}{r}$

each polar equation and convert to rectangular form:

22)  $r = 2 - 2\cos \theta$

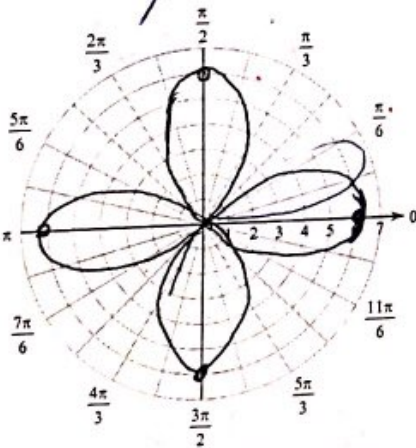
Cardioid



$r = 2 - 2\cos \theta$

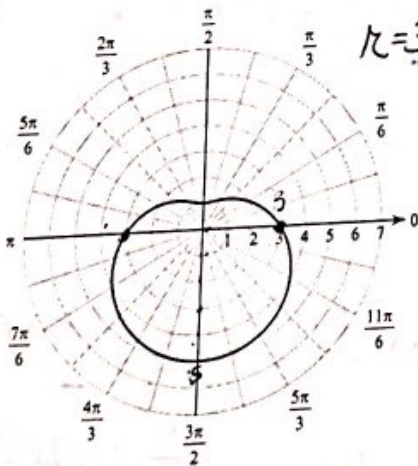
23)  $r = 6\cos(2\theta)$

rose 4pi.

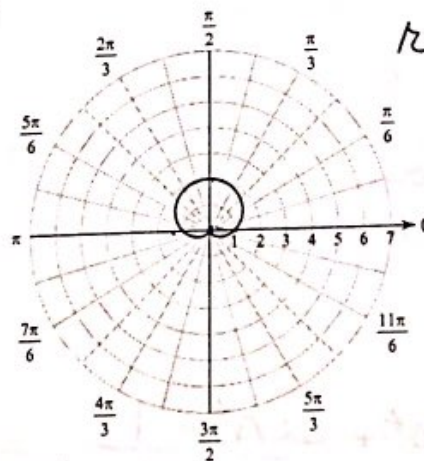


Consider each polar graph. Classify the curve; and write the equation.

24)

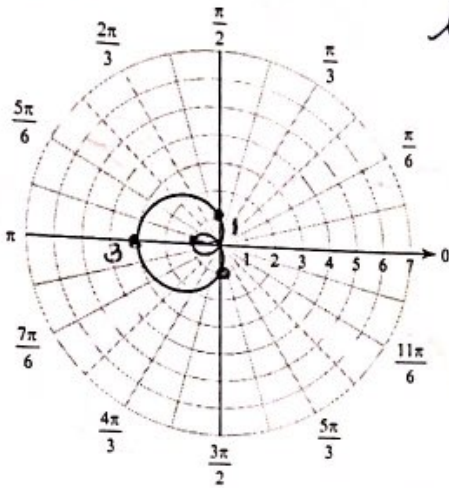


lunicon 25)  
 $r = 3 - 2\cos \theta$



Cardioid  
 $r = 1 + \sin \theta$   
 $r = 1 + \sin \theta$

26)

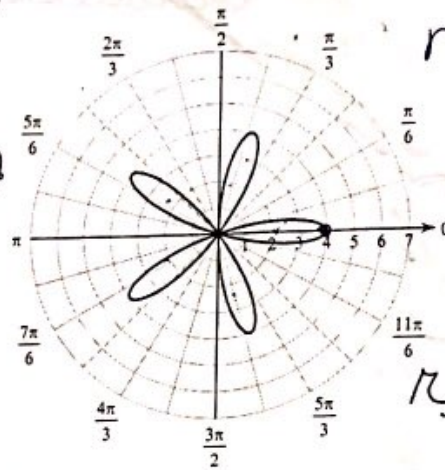


limaçon 27)  
w/loop

$$r = 1 - 2\cos\theta$$

rose

$$r = 4$$



$$r = 4\cos(5\theta)$$

$$4\cos(2\theta)$$

Write each pair of parametric equations in rectangular form.

28)  $x = 5\sin t, y = 2\cos t$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

29)  $x = 3\sin t, y = 3\cos t$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

30)  $x = t + 4, y = \frac{t}{2}$   
 $t = x - 4$       $y = \frac{x-4}{2}$

31)  $x = t - 5, y = 2t^2 - 1$   
 $t = x + 5$       $y = 2(x+5)^2 - 1$   
 $y = 2(x^2 + 10x + 25) - 1$   
 $y = 2x^2 + 20x + 49$

32)  $x = t, y = -\frac{t^2}{6}$

$$y = -\frac{x^2}{6}$$

33)  $x = 2\tan t, y = \sec t$

$$y^2 - \frac{x^2}{4} = 1$$

$$\frac{\cos^2 + \sin^2}{\cos^2} = \frac{1}{\cos^2}$$

$$1 + \tan^2 = \sec^2$$

$$\frac{x}{2} = \tan t$$

$$y = \sec t$$

$$\frac{x^2}{4} = \tan^2$$

$$\frac{y^2}{1} = \sec^2$$

$$\frac{x^2}{4} + \frac{y^2}{1} = \tan^2 + \sec^2$$