

Solve the following trigonometric equations.

1. $3 \sin^2 x - \sin x - 2 = 0$

2. $\tan x \cos^2 x - \tan x = 0$

3. $\cos \theta (\tan \theta - \sqrt{3}) = 0, \quad 0 \leq \theta < 2\pi$

4. $\cot^2 \theta + \cot \theta = 0, \quad 0^\circ \leq \theta < 360^\circ$

5. $\cos 3x = \frac{\sqrt{3}}{2}, \quad 0 \leq \theta < 2\pi$

6. $2 \cos^2 x - \sqrt{3} \cos x = 0, \quad 0 \leq \theta < 2\pi$

7. $4 \sin^2 \left(2\theta - \frac{\pi}{6}\right) = 4, \quad 0 \leq \theta < 2\pi$

8. $\tan \left(x + \frac{\pi}{5}\right) = 1, \quad 0 \leq \theta < 2\pi$

9. $7 \tan x = 2\sqrt{3} + \tan x, \quad 0^\circ \leq \theta < 360^\circ$

10. $3(\sin \beta + 2) = 3 - \sin \beta$

Simplify the following trigonometric expressions.

11. $\csc x \tan x$

12. $\frac{\tan x}{\sec x}$

13. $\frac{\sin^2 x + \tan^2 x + 2 \tan x - 4 + \cos^2 x}{3 \tan^2 x - 3 \tan x}$

14. $\frac{(\sin x + \cos x)(\sin x - \cos x) + 1}{\sin^2 x}$

15. $\frac{\sec^2 y \csc y}{\csc^2 y \sec y}$

16. $\frac{\tan^2 \theta - \sin^2 \theta}{\sec^2 \theta}$

Prove the trig identity.

17. $\frac{\sec x + 1}{\tan x} = \frac{\tan x}{\sec x - 1}$

18. $\frac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$

19. $\sec \alpha - \cos \alpha = \sin \alpha \tan \alpha$

20. $\sin^2 \theta - \tan^2 \theta = -\sin^2 \theta \tan^2 \theta$

21. $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2 \tan x$

22. $\frac{1}{\csc x - \sin x} = \sec x \tan x$