

Find any holes existing in the following functions. Find all asymptotes.

$$1. f(y) = \frac{y^2-16}{y^2-7y+12} = \frac{(y+4)(y-4)}{(y-4)(y-3)} \quad \text{D} (-\infty, 3) \cup (3, 4) \cup (4, \infty)$$

y-inter: $(0, -4/3)$
 H.A: $y=1$
 O.A: none
 hole $(4, 8)$
 VA: $x=3$
 x-inter: $(-4, 0)$

$$2. f(x) = \frac{2x^2+11x+5}{3x^2+17x+10} = \frac{(x+5)(2x+1)}{(x+5)(3x+2)} \quad \text{D} (-\infty, -5) \cup (-5, -2/3) \cup (-2/3, \infty)$$

y-int $(0, 1/2)$
 H.A: $y=2/3$
 O.A: none
 hole $(-5, 9/13)$
 VA: $x=-2/3$
 x-inter $(-1/2, 0)$

$$3. f(x) = \frac{3x^2-3}{6x^2+12x+6} = \frac{3(x+1)(x-1)}{6(x+1)(x+1)} \quad \text{D} (-\infty, -1) \cup (-1, \infty)$$

y-int $(0, -1/2)$
 H.A: $y=1/2$
 O.A: none
 hole $(-1, \text{undefined})$
 V.A. bumps hole
 x-inter $(1, 0)$
 V.A: $x=-1$

$$4. f(x) = \frac{-15-x+6x^2}{3x^2+4x-15} = \frac{6x^2-x-15}{(x+3)(3x-5)}$$

y-int $(0, 1)$
 HA: $y=2$
 O.A: none
 hole $(5/3, 5/7)$
 x-int $(-3/2, 0)$
 VA: $x=-3$
 D $(-\infty, -3/2) \cup (-3/2, 5/3) \cup (5/3, \infty)$

$$5. f(x) = \frac{-4x^2-6x+54}{-2x^2-3x+27} = \frac{-2(2x+9)(x-3)}{(2x+9)(2x-9)}$$

H.A: $y=-1$
 y-int $(0, -2/3)$
 O.A: none
 hole $(-9/2, -5/6)$
 x-int: $(3, 0)$
 D $(-\infty, -9/2) \cup (-9/2, 9/2) \cup (9/2, \infty)$

$$6. f(x) = \frac{6x^2+x+12}{3x^2-5x-2} = \frac{6x^2+x+12}{(3x+2)(x-1)}$$

holes none
 x-int none
 VA: $x=-1/3, x=2$
 D $(-\infty, -1/3) \cup (-1/3, 2) \cup (2, \infty)$

$$7. f(x) = \frac{x^2+3x}{x^3+2x^2+x} = \frac{x(x+3)}{x(x+1)(x+1)}$$

y-int: none
 H.A: $y=0$
 O.A: none
 Hole $(0, 3)$
 x-int $(-3, 0)$
 VA: $x=-1$
 D $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$