Precalculus
Vector Test Review

Name Key
Date

For 1-3, determine the direction and magnitude of the vector $\overrightarrow{PQ}$

1. $P = (2,3),\ Q = (5,9)$

$V_{PQ} = \langle 3, 6\rangle$

$||v|| = \sqrt{3^2+6^2}$

$= \sqrt{45} = 3\sqrt{5}$

2. $P = (-7,0),\ Q = (-4,-5)$

$V_{PQ} = \langle 3, -5\rangle$

$||v|| = \sqrt{9+25}$

$= \sqrt{34}$

3. $P = (-3,5),\ Q = (7,-11)$

$V_{PQ} = \langle 10, -16\rangle$

$||v|| = \sqrt{100+256}$

$= \sqrt{356} = 2\sqrt{89}$

For 4-10, let $u = \langle 3, 1\rangle$, $v = 2i + j$, and $w = \langle -6, -2\rangle$. Evaluate. Write any resultant vectors in $ai + bj$ form.

4. $u + v$

$5i + 2j$

5. $u - v$

i

6. $2(v - w)$

$2\langle 8, 3\rangle$

$16i + 6j$

7. $u + \frac{1}{2}w$

$\langle 3,1\rangle + \langle -3, -1\rangle$

0

8. $u \cdot v$

$3(2) + 1(1)$

7

9. $w \cdot v$

$-6(2) + -2(1)$

-14

10. $u \cdot w$

$3(-6) + 1(-2)$

-20

11. Unit vector of w

$||w|| = \sqrt{40} = 2\sqrt{10}$

$\left\langle \frac{-6}{2\sqrt{10}}, \frac{-2}{2\sqrt{10}} \right\rangle = \left\langle \frac{-3\sqrt{10}}{10}, \frac{-\sqrt{10}}{10} \right\rangle$

$\frac{-3\sqrt{10}}{10}i - \frac{\sqrt{10}}{10}j$

For 12-14, determine the component form of the vector v whose magnitude and direction angle θ are given.

$||v||(\cos\theta i + \sin\theta j)$

12. $||v|| = 10, \theta = 225°$

$10(\cos 225 i + \sin 225 j)$

$10\left(\frac{-\sqrt{2}}{2}\right) + 10\left(\frac{-\sqrt{2}}{2}\right)$

$\langle -5\sqrt{2}, -5\sqrt{2}\rangle$

13. $||v|| = 3, \theta = 310°$

$\langle 3\cos 310, 3\sin 310\rangle$

$\langle 1.928, -2.298\rangle$

14. $||v|| = 6, \theta = 140°$

Exact

$\langle 6\cos 140, 6\sin 140\rangle$

$\langle -4.596, 3.857\rangle$

For 15-17, determine the angle between vectors u and v.

$\cos\theta = \frac{u \cdot v}{||u||\,||v||}$

15. $u = \langle 2, 4\rangle,\ v = \langle 0, -5\rangle$

$u \cdot v = 0 + -20 = -20$

$||u|| = \sqrt{4+16} = \sqrt{20}$

$||v|| = \sqrt{0+25} = 5$

$\cos\theta = \frac{-20}{5\sqrt{20}}$

$\theta = \cos^{-1}\left(\frac{-20}{5\sqrt{20}}\right)$

$\theta \approx 153.435°$

16. $u = 2j,\ v = 4i + j$

$u \cdot v = 0(4) + 2(1) = 2$

$||u|| = \sqrt{0+4} = 2$

$||v|| = \sqrt{16+1} = \sqrt{17}$

$\cos\theta = \frac{2}{2\sqrt{17}}$

$\theta \approx 75.964$

17. $u = \sqrt{2}i + \sqrt{2}j,\ v = i - j$

$u \cdot v = \sqrt{2}(1) + \sqrt{2}(-1) = 0$

$||u|| = \sqrt{2+2} = 2$

$||v|| = \sqrt{1+1} = \sqrt{2}$

$\cos\theta = \frac{0}{2\sqrt{2}}$

$\theta = 90°$

For 18-20, write the resultant component vector in $v = ai + bj$ form.

18. A vector with a magnitude of 60 and a bearing of 199°. $\theta = 251°$

$60(\cos 251° i + \sin 251° j)$

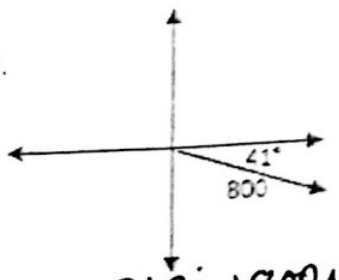
$60\cos 251 i + 60\sin 251 j$

19. A vector with a length of 5 and a direction of N81°W.

$5(\cos 171 i + \sin 171 j)$

$= 5\cos 171 i + 5\sin 171 j$

20.



$800\cos 319 i + 800\sin 319$

21. Jane left camp three days ago on a journey into the jungle. Three days of her journey can be described by position vectors $\overrightarrow{d_1} = \langle 7, 8\rangle$, $\overrightarrow{d_2} = \langle 6, 2\rangle$, and $\overrightarrow{d_3} = \langle 2, 9\rangle$. How far is Jane from the camp at the end of day three and in what direction is Jane from camp at the end of day three? Assume 0° is the rightward direction.

$\langle 7+6+2, 8+2+9\rangle$

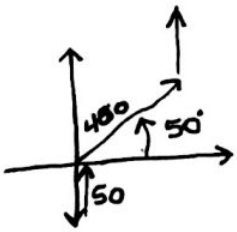
$\langle 15, 19\rangle$

$\sqrt{15^2+19^2} \approx 24.207$

$\tan^{-1}\left(\frac{19}{15}\right) \approx 51.710°$

she is 24.207 units from camp in a CCW $\angle = 51.710°$

22. A plane traveling at 400 mph is flying with a bearing of 40°. There is a wind of 50 mph from the south. If no correction is made for the wind, what is the final bearing and speed of the plane?



$V_{plane}: 400\cos 50 i + 400\sin 50 j$ $V_{wind}: 50\cos 90 i + 50\sin 90 j$

$V_{p+w} = (400\cos 50 + 50\cos 90) i + (400\sin 50 + 50\sin 90) j$

(x) (y)

$\|v_{p+w}\| = \sqrt{x^2+y^2} \approx 439.479$ mph

$\tan^{-1} = \left(\frac{y}{x}\right) \approx 54.194°$

* Plane flying 439 mph bearing 36° (N36E)

23. An object at the origin is acted upon by two forces. A 150-pound force makes an angle of 20° with the positive x-axis, and the other force of 100 pounds makes an angle of 70° with the positive x-axis. Determine the direction and magnitude of the resultant force.

$V_1 = 150\cos 20 i + 150\sin 20 j$ $V_2 = 100\cos 70° i + 100\sin 70 j$

$V_{1+2} = (150\cos 20 + 100\cos 70) i + (150\sin 20 + 100\sin 70) j$

(x) (y)

$\|V_{1+2}\| = \sqrt{x^2+y^2} = 227.560$ pounds

$\tan^{-1}\left(\frac{y}{x}\right) \approx 39.672°$

The resultant force is 228 pounds @ $\angle$ of 40° with positive x axis

d the ba' Eliminate the parameter.

24. $x = t - 3, \quad y = 2t + 1, \quad t \geq 0$

$t = x+3$

$y = 2(x+3)+1$

$= 2x+6+1$

$\boxed{y = 2x+7}$

25. $x = t + 5, \quad y = \sqrt{t}, \quad t \geq 0$

$t = x-5$

$\boxed{y = \sqrt{x-5}}$

26. $x = -2 + t^2, \quad y = 1 + 2t^2, \quad$ *for any t*

$t^2 = x+2 \qquad y = 1+2(x+2)$

$t = \sqrt{x+2} \qquad y = 1+2x+4$

$\boxed{y = 2x+5}$

27. $x = e^t, \quad y = t, \quad$ *for any t*

$\ln x = t$

$\boxed{y = \ln x}$

28. $x = 3\cos t, \quad y = 3\sin t, \quad 0 \leq t \leq 2\pi$

$\cos t = x/3 \qquad \text{sen}\, t = y/3$

$\boxed{\frac{x^2}{9} + \frac{y^2}{9} = 1}$

29. $x = 2\sin t - 3, \quad y = 2\cos t + 1, \quad 0 \leq t \leq 2\pi$

$\frac{x+3}{2} = \text{sen}\, t \qquad \frac{y-1}{2} = \cos t$

$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{4} = 1$

30. A golfer at a driving range stands on a platform 2 feet above the ground and hits the ball with an initial velocity of 120 feet/second at an angle of 39° with the horizontal. There is a 32-foot-high fence 400 feet away. Determine the position of the ball as a pair of parametric equations. Will the ball fall short, hit the fence, or go over the wall?

$x = 120\cos 39t \qquad y = -16t^2 + 120\,\text{sen}\,39t + 2$

$h = 2$ ft

$V_0 = 120$ ft/sec

$39°$

The Ball will hit the Wall

1st when is the ball 400 feet away

$400 = 120\cos 39t$

$\frac{400}{120\cos 39} = t \qquad t \approx 4.289$ sec

2nd: how high is the ball at 4.289 sec

$y = -16(4.289)^2 + 120\,\text{sen}(39)(4.289) + 2 \approx 3$

31. Suppose a professional football player kicks a football with an initial velocity of 29 yards per second at an angle of 68° to the horizontal. Suppose a kick returner catches the ball 5 seconds later. Determine the position of the ball as a pair of parametric equations. How far has the ball traveled horizontally and what is its vertical height at the time it is caught?

$x = 87\cos 68t \qquad y = -16t^2 + 87\,\text{sen}\,68t + 0$

$h = 0$

$V_0 = 29(3) = 87$ ft

$\theta = 68°$

$x = 87\cos 68(5) \approx 162.954$

$y = -16(5)^2 + 87\,\text{sen}(68)(5) + 0 \approx 3.325$

The ball will travel 54.3 yards and was ~ 1.1 yards off the ground when caught.