Key

Unit 2 Test Review

For each of the following, find all of the intercepts, the domain & range, the local and absolute extrema, and the increasing, decreaing and constant intervals.

1) $f(x) = x^4 - 4x^2 + 2x + 2$ on the interval $(-\infty, \infty)$

Inc $(-1.526, 0.259) \cup (1.267, \infty)$
dec $(-\infty, -1.526) \cup (.259, 1.267)$
absolute min $(-1.52, -4.94)$
local min ~~$(-1.52, -4.94)$~~ $(1.27, .69)$
x-int $(-2.12, 0)(-5.11, 0)$
y-int $(0, 2)$
Domain $(-\infty, \infty)$ Range $[-4.944, \infty)$ Local Max $(.258, 2.2$

2) $f(x) = x^4 - 4x^2 + 2x + 2$ on the interval $(-2, 1)$

x-int $(-.511, 0)$
y-int $(0, 2)$
Domain $(-2, 1)$
local/absolute min $(-1.526, -4.944)$
local/absol Max $(.259, 2.254)$
Increase $(-1.526, .259)$
decrease $(-2, -1.526) \cup (.259, 2)$

Determine the domain, range, intercepts, holes, asymptotes, extrema, increasing and decreasing intervals for each of the following functions. Only use calculator for extrema & intervals.

3) $f(x) = \frac{2x^2 - 13x + 15}{2x^3 - 7x^2 + 6x}$.

See back page

y-in: none
H.A
O.A:
D(
hole:
x int: (
VF

4) $g(x) = \frac{3x^3 - x^2 - 4x}{6x^2 + 5x - 6} = \frac{x(3x-4)(x+1)}{(2x+3)(3x-2)}$

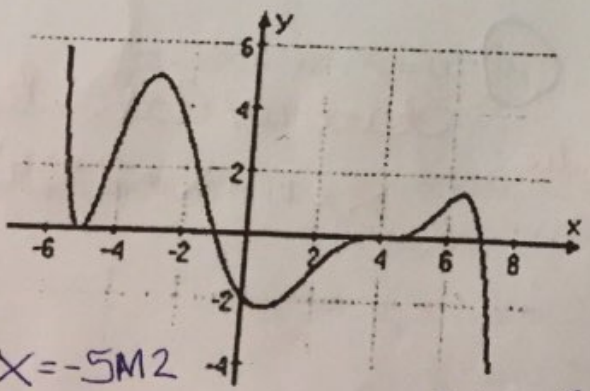
y int $(0, 0)$
HA: none
O.A $y = \frac{1}{2}x - \frac{7}{36}$

$$\begin{array}{r|l} 6x^2+5x-6 & 3x^3 - x^2 - 4x \\ & \underline{3x^3 + \frac{5}{2}x^2} \\ & -\frac{7}{2}x^2 \end{array}$$

hole: none
x int $(0,0)(4/3, 0)(-1, 0)$
dom $(-\infty, -3/2) \cup (-3/2, 2/3) \cup (2/3, \infty)$
R $(-\infty, \infty)$

Write a linear factorization for the following graphs.

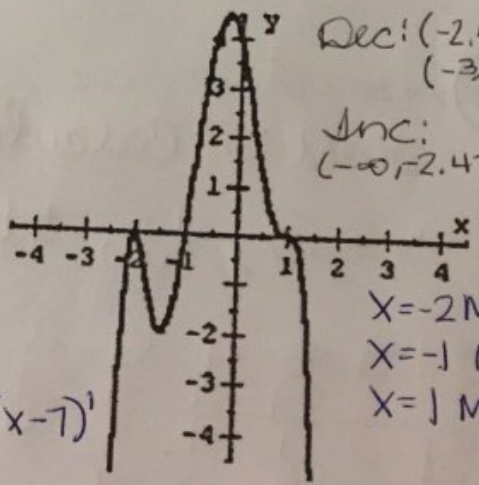
5)



X = -5 M 2
X = -1
X = 4 M 3
X = 7 M 1

$y = -(x+1)(x+5)^2(x-4)^3(x-7)^1$

6)



Dec: $(-2.47, -3/2) \cup (-3/2, -0.6)$
Inc: $(-\infty, -2.47) \cup (-.6, 2/3) \cup (2/3, \infty)$
X = -2 M 2
X = -1 M 1
X = 1 M 3

$y = -(x+2)^2(x+1)(x-1)^3$

Find all real and complex roots using the method of your choice.

7) $y = x^5 - x^2$

$= x^2(x^3 - 1)$

$= x^2(x-1)(x^2+x+1)$

$x = 0; 1; \frac{-1 \pm i\sqrt{3}}{2}$

8) $f(x) = x^3 - 3x^2 + 6x - 18$

grouping

$= (x^2+6)(x-3)$

$x = \pm i\sqrt{6}$ $x = 3$

9) $g(x) = x^4 - 16$

$= (x^2+4)(x^2-4)$

$= x = \pm 2i$ $x = \pm 2$

10) $y = 2x^2 + 3x - 9$

$= (x+3)(2x-3)$

$= x = -3; 3/2$

$(7x+2)(49x^2 - 14x + 4)$

$2(4x^2 - 2x - 9)$

11) $f(x) = 343x^3 + 8$

$x = -2/7$ $x = \frac{1 \pm i\sqrt{3}}{2}$

12) $h(x) = 8x^2 - 4x - 18$

$= \frac{1 \pm \sqrt{37}}{4}$

13) $y = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

Start w/ Calculator

$x = -2; -1/2; \pm i$

synthetic

-2 |	2	5	4	5	2
		-4	-2	-4	-2
$-1/2$ |	2	1	2	1	0
		-1	0	-1	
	2	0	2	0	

$x^2 + 1 = 0$

$x = \pm i$

14) $f(x) = x^3 - 8x^2 + 29x - 52$

Start w/ Calculator

$x = 4$ $x = 2 \pm 3i$

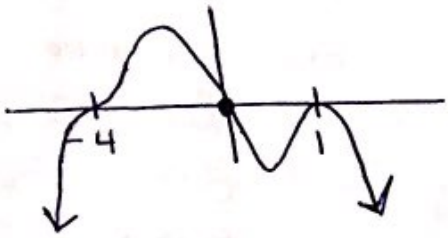
Sketch a graph a graph for each of the following.

15) $y = -x(x+4)^3(x-1)^2$

$\lim_{x\to\infty} f(x) = -\infty$

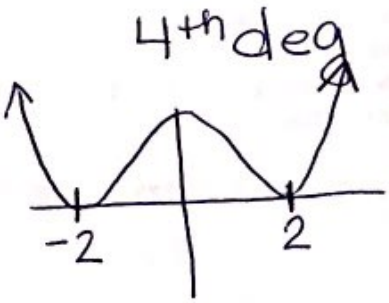
$\lim_{x\to-\infty} f(x) = -\infty$

6th deg



16) $f(x) = 3(x^2-4)^2$

4th deg



$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

Write the equation of the polynomial in standard form that has the given roots in standard form.

17) A polynomial with a degree of 3 and root of 3 and $4-i$.

$x=3 \quad x=4-i \quad x=4+i$

$y = x^3 - 11x^2 + 41x - 51$

18) A polynomial with a degree of 4 and root of -2 with a multiplicity of 2 and a root of $-2i$.

$x=-2 \quad x=-2 \quad x=-2i \quad x=2i$

$y = x^4 + 4x^3 + 8x^2 + 16x + 16$

19) A polynomial with a degree of 2 and a root of $1-3\sqrt{2}$

$x = 1-3\sqrt{2} \quad x = 1+3\sqrt{2}$

$y = (x-1+3\sqrt{2})(x-1-3\sqrt{2})$

$x^2 - x - 3x\sqrt{2} - x + 1 + 3\sqrt{2} + 3x\sqrt{2} - 3\sqrt{2} + 9\sqrt{4}$

$y = x^2 - 2x + 1 - 18$

$y = x^2 - 2x - 17$

Operations With Rational Expressions----------KEY POINTS TO REMEMBER

* ALWAYS factor 1ST!!!!!!
* You DO NOT need LCD when multiplying and dividing
* Remember how you work with regular fractions to add them, follow the SAME process
  * When adding/subtracting DO NOT cancel the factors YOU multiplied in to make the LCD before you add/subtract

Perform each of the following operations, write your answer in the SIMPLIEST form possible, & state the restrictions.

20) $\frac{6x^2-7x-3}{8x^2-2x-15}$ $= \frac{3x+1}{4x+5}$

$\frac{(2x-3)(3x+1)}{(2x-3)(4x+5)}$

20) $\frac{x^2+16x+55}{x^2-8x-65} \cdot \frac{x^3-11x^2-26x}{x^2+13x+22}$

$\frac{(x+11)(x+5)}{(x-13)(x+5)} \cdot \frac{x(x-13)(x+2)}{(x+11)(x+2)}$

$= x$

21) $\frac{x^4-1}{x^3-3x^2+x-3} \cdot (4x^2-7x-15)$

$\frac{(x^2+1)(x+1)(x-1)}{(x^2+1)(x-3)} \cdot \frac{(x-3)(4x+5)}{1}$

$= (x^2-1)(4x+5)$

22) $\frac{x^3+64}{x^2-16} \div (x^2-8x+16)$

$\frac{(x^2-4x+16)}{(x-4)^3}$

23) $\frac{5x^2}{2x^2+5x-33} \div \frac{5x^3-20x}{2x^2+15x+22}$

$\frac{5x^2}{(2x+11)(x-3)} \cdot \frac{(2x+11)(x+2)}{5x(x+2)(x-2)}$

$\frac{x}{(x-3)(x-2)}$

24) $\frac{x-1}{x^2-17x+72} - \frac{x}{x^2-3x-54}$

$(x-9)(x-8) \quad (x+6)(x-9)$

$\frac{(x-1)(x+6) - x(x-8)}{(x-9)(x-8)(x+6)}$

$\frac{13x-6}{(x-9)(x-8)(x+6)}$

25) $\frac{\frac{x}{3}+5}{7+\frac{6}{x}}$ $\frac{x(x+15)}{3(7x+6)}$

$= \frac{\frac{x+15}{3}}{\frac{7x+6}{x}}$

$= \frac{x+15}{3} \cdot \frac{x}{7x+6} =$

26) $\frac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}}$ $= \frac{5(x+1)}{3(x-1)}$

$\frac{\frac{5}{x-2}}{\frac{1(x+1)+2(x-2)}{(x-2)(x+1)}}$

$= \frac{5}{x-2} \cdot \frac{(x-2)(x+1)}{3x-3}$

$3(x-1)$

27) $\frac{\frac{(x+y)^3}{x^2-y^2}}{\frac{(x^2+2xy+y^2)}{x^3-y^3}}$

$= \frac{(x+y)(x+y)(x+y)}{(x+y)(x-y)} \cdot \frac{(x-y)(x^2+xy+y^2)}{(x+y)(x+y)}$

$= x^2+xy+y^2$

Solving Rational Equations----------KEY POINTS TO REMEMBER

* Factor 1st & Simplify if you can
* Find the LCD & multiply EVERY SINGLE term in the equation by that LCD to clear ALL fractions.
* CHECK EVERY ANSWER!!!!

28) $\frac{x-2}{x+4}+\frac{x+1}{x+6}=\frac{11x+32}{x^2+10x+24}$ $(x+6)(x+4)$ $x=5, \cancel{-4}$ Extran

$(x-2)(x+6)+(x+1)(x+4)=11x+32$

$x^2+4x-12+x^2+5x+4=11x+32$

$2x^2-2x-40=0$

$x^2-x-20=0 \Rightarrow (x-5)(x+4)=0$

29) $\frac{x+3}{x+2}=1-\frac{x+1}{x+2}$

$x+3=1(x+2)-x-1$

$x+3=x+2-x-1$

$x=\cancel{-2}$ Extran $\emptyset$

30) $\frac{3x}{x-7}-\frac{1}{x-2}=\frac{5}{x^2-9x+14}$ $(x-7)(x-2)$

$3x(x-2)-1(x-7)=5$

$3x^2-6x-x+7=5$

$3x^2-7x+2=0$ (6, 1)

$(x-2)(3x-1)=0$

$\cancel{x=2}$ Extran

$x=1/3$

31) $\frac{2}{x+3}+\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$ $-1(x-4)$ $(x-4)(x+3)$

$2(x-4)+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

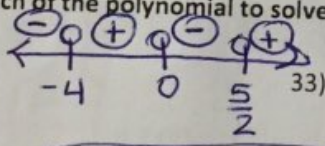
$3x=-3$

$x=-1$

Solving Rational & Polynomial Inequalities----------KEY POINTS TO REMEMBER

* Factor 1st & Simplify if you can
* If there are rational expressions on BOTH sides you MUST move them to ONE side
* You CANNOT EVER multiply an INEQUALITY by the LCD on both sides
* When using a SIGN CHART SHOW ALL WORK!!!
* If using a sketch of the polynomial to solve, then you MUST CLEARLY show the LABELED SKETCH

32) $\frac{3}{4}+\frac{x}{2}>\frac{5}{x}$



$\frac{3}{4}+\frac{x}{2}-\frac{5}{x}>0$

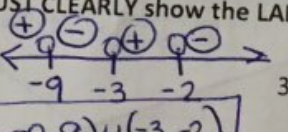
$\frac{3x+2x^2-20}{4x}>0$ (40, 8, -5)

$\frac{2x^2+3x-20}{4x}>0$

$\frac{(x+4)(2x-5)}{4x}>0$

$(-4,0)\cup(5/2,\infty)$

33) $\frac{6}{x+3}>x+8$



$\frac{6}{x+3}-x-8>0$

$\frac{6-x(x+3)-8(x+3)}{x+3}>0$

$\frac{6-x^2-3x-8x-24}{x+3}>0$

$\frac{-x^2-11x-18}{x+3}>0$

$\frac{-1(x^2+11x+18)}{x+3}>0$

$\frac{-1(x+9)(x+2)}{x+3}>0$

$(-\infty,-9)\cup(-3,-2)$

34) $\frac{8}{x-3}+\frac{8}{x+1}\geq -3$

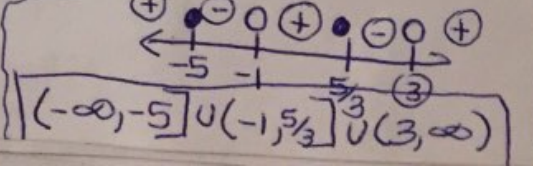
$\frac{8(x-3)+8(x+1)+3(x-3)(x+1)}{(x-3)(x+1)}\geq 0$

$\frac{8x-24+8x+8+3(x^2-2x-3)}{(x-3)(x+1)}\geq 0$

$\frac{16x-16+3x^2-6x-9}{(x-3)(x+1)}\geq 0$

$\frac{3x^2+10x-25}{(x-3)(x+1)}\geq 0$ (25, -5, 15/3)

$\frac{(3x-5)(x+5)}{(x-3)(x+1)}\geq 0$



$(-\infty,-5]\cup(-1,5/3]\cup(3,\infty)$

$[3,4) \cup (4,\infty)$

35) $\frac{10}{x-4} + x \geq \frac{3x-2}{x-4}$

$\frac{10}{x-4} + x - \left(\frac{3x-2}{x-4}\right) \geq 0$

$\frac{10 + x(x-4) - 3x + 2}{x-4} \geq 0$

36) $(2x-4)(x-3) > 0$

$(-\infty, 2) \cup (3, \infty)$

37) $\frac{(x-1)^2}{(x+1)(x+2)} > 0$

$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$

38) $\frac{x^2-4}{x^2+4} \geq 0$

39) $\frac{3+x}{3-x} \geq 1$

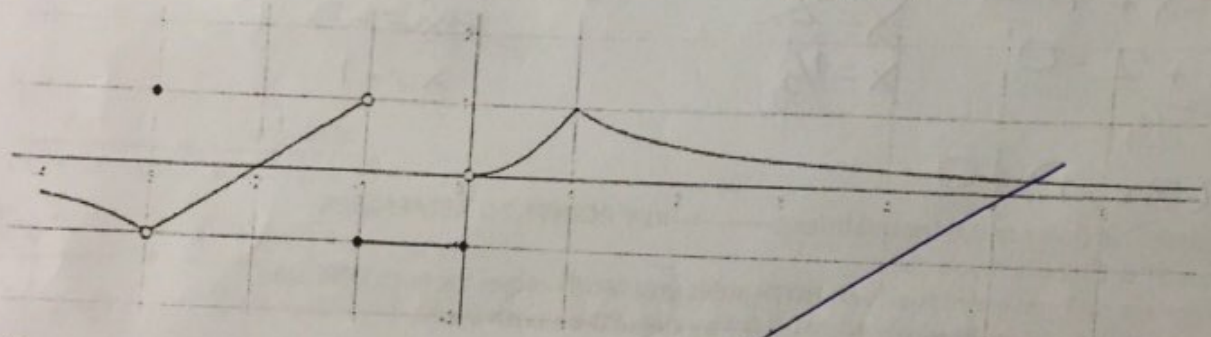
40) $x^3 + 9x^2 + 20x + 12 < 0$

~~$x^2(x+9) + 4(5x+3)$~~

See last page

Omit

41) Determine the limits and evaluate the function at the given value.



A) $\lim_{x \to -3^-} f(x) =$

B) $\lim_{x \to -3^+} f(x) =$

C) $\lim_{x \to -3} f(x) =$

D) $f(-3) =$

E) $\lim_{x \to -2^-} f(x) =$

F) $\lim_{x \to -2^+} f(x) =$

G) $\lim_{x \to -2} f(x) =$

H) $f(-2) =$

$\lim_{x \to -1^-} f(x) =$

J) $\lim_{x \to -1^+} f(x) =$

K) $\lim_{x \to -1} f(x) =$

L) $f(-1) =$

) $\lim_{x \to 0^-} f(x) =$

N) $\lim_{x \to 0^+} f(x) =$

O) $\lim_{x \to 0} f(x) =$

P) $f(0) =$

Graphing Rational Functions----------KEY POINTS TO REMEMBER

* ALWAYS factor 1ST, if the expression simplifies there is a HOLE in the graph.
* If there is a hole in the graph, ALL FURTHER CALCULATIONS should be done using the SIMPLIFIED expression
* Real zeros of the numerator (using the SIMPLIFIED version) are x-intercepts of the function
* Real zeros of the denominator (using the SIMPLIFIED version) are the locations of the VERTICAL ASYMPTOTES
* Look at your notes for the THREE situations to determine the end behavior asymptotes of the function
* Look at your notes or the textbook to determine what the following notation means
* Asymptotes are written as EQUATIONS (look in your notes/textbook to clarify)

42) Determine the holes, intercepts, asymptotes, and then sketch each of the following:

a) $f(x) = \frac{x^2-4}{x^2-9} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$

Hole(s): none

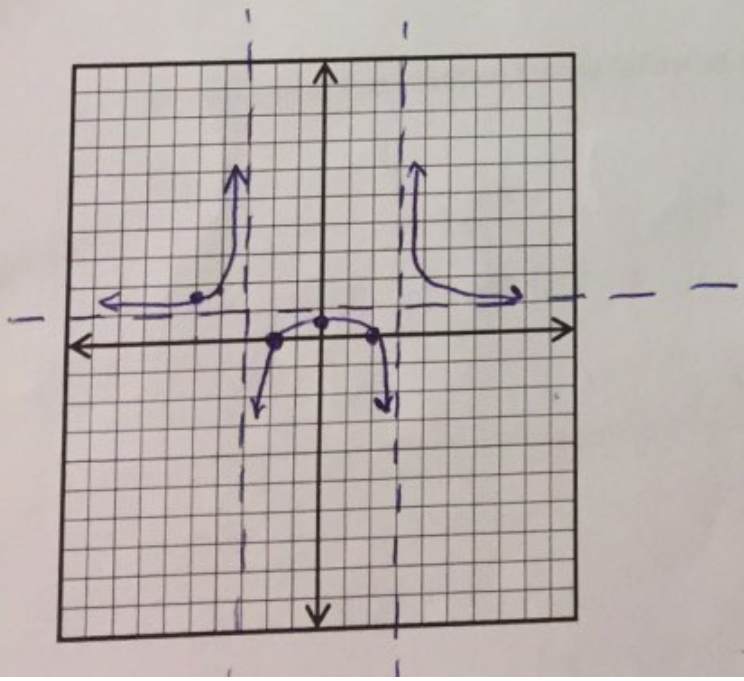
x-int: (2, 0) (-2, 0)

y-int: (0, 4/9)

Eqs of ALL Asymptotes: HA: $y=1$ VA: $x=3$, $x=-3$

State End Behavior

$\lim_{x\to\infty} f(x) = 1$ $\lim_{x\to-\infty} f(x) = 1$



b) $f(x) = \frac{3x^2-x-4}{9x^3+9x^2-16x-16} = \frac{(3x-4)(x+1)}{(x-1)(3x+4)(3x-4)} = \frac{x+1}{(x-1)(3x+4)}$

Hole(s): (4/3, 7/8) (__, __)

x-int: (-1, 0) (__, __) (__, __)

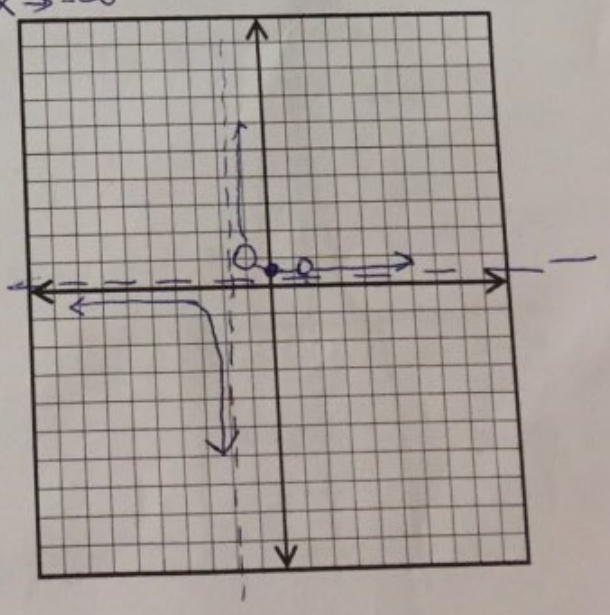
y-int: (0, 1/4)

next page

Eqs of ALL Asymptotes: HA $y=0$ VA $x=1$, $x=-4/3$

State End Behavior

$\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = 0$



(3) $\dfrac{2x^2-13x+15}{2x^3-7x^2+6x} = \dfrac{(x-5)(2x-3)}{x(x-2)(2x-3)}$

y-int none

H.A $y=0$

O.A none

$f(x) = \dfrac{(x-5)(2x-3)}{x(x-2)(2x-3)}$

D $(-\infty,0)\cup(0,3/2)\cup(3/2,2)\cup(2,\infty)$

hole: $(3/2, 14/3)$ $\quad y = \dfrac{x-5}{x(x-2)}$

X-int $(5,0)$

V.A $x=0$ $x=2$

H.A: Intersect $0 = \dfrac{x-5}{x(x-2)}$

$x-5=0$

$x=5$

Local Min $(1.13, 3.94)$

Dec $(-\infty,0)\cup(0,1.13)$

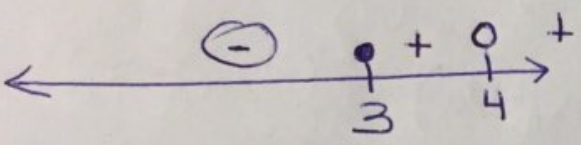
Inc $(1.13,1.5)\cup(1.5,2)\cup(2,\infty)$

35 continued

$$\frac{10 + x^2 - 4x - 3x + 2}{x-4} \geq 0$$

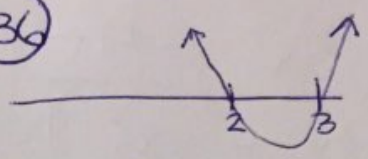
$$\frac{x^2 - 7x + 12}{x-4} \geq 0$$

$$\frac{(x-3)(x-4)}{x-4} \geq 0$$



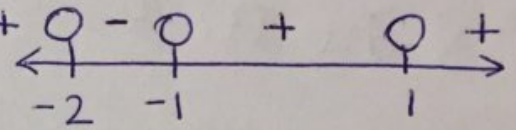
$[3,4) \cup (4,\infty)$

36



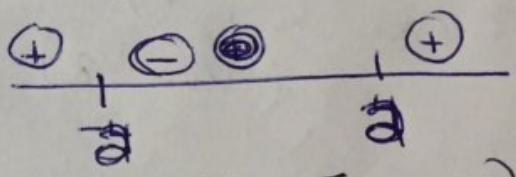
$(-\infty, 2) \cup (3, \infty)$

37



$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$

38 $\frac{(x+2)(x-2)}{x^2+4} \geq 0$



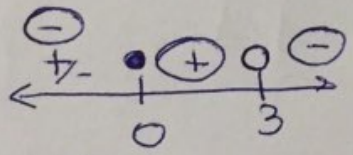
$(-\infty, -2] \cup [2, \infty)$

39 $\frac{x+3}{-1(x-3)} - 1 \geq 0$

$$\frac{-x-3 \quad -1(x-3)}{x-3} \geq 0$$

$$\frac{-x-3-x+3}{x-3} \geq 0$$

$$\frac{-2x}{x-3} \geq 0$$



$[0, 3)$

12b) $f(x) = \dfrac{3x^2 - x - 4}{9x^3 + 9x^2 - 16x - 16}$

H.A $y = 0$

Y-int $(0, 1/4)$

$f(x) = \dfrac{(3x-4)(x+1)}{(x+1)(3x+4)(3x-4)}$ $\quad x \neq -1, -4/3, 4/3$

D $(-\infty, -4/3) \cup (-4/3, -1) \cup (-1, 4/3) \cup (4/3, \infty)$

hole $(-1, 1)$ $\quad f(x) = \dfrac{1}{3x+4}$

$(4/3, 1/8)$

x-int none

VA $x = -4/3$