

PreCalculus
Unit 5: Analytic Trig Review
Solve each equation.

Name Key Date _____

1. $4 \cos^2 x - 3 = 0$

$x = \pi/6 \pm 2\pi k$ $x = \frac{11\pi}{6} \pm 2\pi k$
 $x = 5\pi/6 \pm 2\pi k$ $x = \frac{7\pi}{6} \pm 2\pi k$

2. $2 \sin^2 x - \sin x - 3 = 0, \quad 0 \leq x < 2\pi$

$x = \frac{3\pi}{2}$

3. $\sin(3\theta) = -1, \quad 0 \leq \theta < 2\pi$

$\theta = \pi/2, 7\pi/6, 11\pi/6$

4. $\sqrt{3} + \tan(2\theta) = 0 \quad 0 \leq x < 2\pi$

$\theta = \pi/3, 5\pi/6, 8\pi/6, 11\pi/6$

5. $\sin\left(\frac{\theta}{2}\right) - 1 = 0$

$\theta = \pi + 4\pi k$

6. $\sec\left(x - \frac{\pi}{5}\right) = 2, \quad 0 \leq x < 2\pi$

$\theta = \frac{8\pi}{15}, \frac{28\pi}{15}$

7. $4 \cos^2 x + 4 \cos x - 3 = 0, \quad 0 \leq x < 2\pi$

$x = \pi/3, 5\pi/3$

8. $2 \tan^2 x - \tan x - 6 = 0, \quad 0 \leq x < 2\pi$

$x = 2.159, 5.300$
 $x = 1.107, 4.249$

9. $\sin \theta = -0.2, \quad 0 \leq \theta < 2\pi$

$\theta = 6.082, 3.343$

10. $\tan x = 5, \quad 0 \leq x < 2\pi$

$x = 1.373, 4.515$

Determine the exact value of each trigonometric function using a sum/difference and then again using a half/double angle to verify.

11. $\cos \frac{7\pi}{12}$

$\cos(135-30)$
 $\frac{-\sqrt{6} + \sqrt{2}}{4}$

half \angle
 $\frac{-\sqrt{2-\sqrt{3}}}{2}$

12. $\tan\left(-\frac{\pi}{12}\right)$

sum/diff
 $-2 + \sqrt{3}$

half/double
 $-2 + \sqrt{3}$

13. $\sin \frac{5\pi}{12}$

sum/diff
 $\frac{\sqrt{6} + \sqrt{2}}{4}$

half/double
 $\frac{\sqrt{2+\sqrt{3}}}{2}$

Prove each trigonometric identity.

14. $\tan \theta \cot \theta - \sin^2 \theta = \cos^2 \theta$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} - \sin^2 \theta =$$

$$1 - \sin^2 \theta$$

$$\cos^2 \theta = \cos^2 \theta$$

15. $4\cos^2 \theta + 3\sin^2 \theta = 3 + \cos^2 \theta$

$$4\cos^2 \theta + 3(1 - \cos^2 \theta)$$

$$4\cos^2 \theta + 3 - 3\cos^2 \theta$$

$$\cos^2 \theta + 3 = 3 + \cos^2 \theta$$

16. $\frac{\csc \theta}{1 + \csc \theta} = \frac{1 - \sin \theta}{\cos^2 \theta}$

$$\frac{\frac{1}{\sin \theta}}{1 + \frac{1}{\sin \theta}} = \frac{1}{\sin \theta + 1}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$= \frac{1 - \sin \theta}{1 - \sin^2 \theta}$$

$$= \frac{1 - \sin \theta}{\cos^2 \theta}$$

17. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2$

$$= \tan^2 \theta (\tan^2 \theta + 1)$$

$$= \tan^2 \theta (\sec^2 \theta)$$

$$= (\sec^2 \theta - 1)(\sec^2 \theta)$$

$$= \sec^4 \theta - \sec^2 \theta$$

18. $\frac{\csc(\sin \theta + \cos \theta)}{\sin \theta} \cdot \frac{\sec(\cos \theta - \sin \theta)}{\cos \theta} = \sec \theta \csc \theta$

$$\frac{\sin \theta \csc \theta + \cos^2 \theta - \sin \theta \csc \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta}$$

$$= \sec \theta \csc \theta = \sec \theta \csc \theta$$

19. $\sin(\alpha - \beta) \sin(\alpha + \beta) = \sin^2 \alpha - \sin^2 \beta$

$$(\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\sin \alpha \cos \beta + \cos \alpha \sin \beta) =$$

$$\sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta$$

$$\sin^2 \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \sin^2 \beta$$

$$\sin^2 \alpha - \sin^2 \alpha \sin^2 \beta - \sin^2 \beta + \sin^2 \beta \sin^2 \alpha$$

$$\sin^2 \alpha - \sin^2 \beta = \sin^2 \alpha - \sin^2 \beta$$

20. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$2 \cos \alpha \cos \beta = 2 \cos \alpha \cos \beta$$

21. $\frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cot \theta - 1}{\cot \theta + 1}$

$$\frac{\cos \theta - 1}{\sin \theta} = \frac{\cos \theta - \sin \theta}{\sin \theta}$$

$$\frac{\cos \theta + 1}{\sin \theta} = \frac{\cos \theta + \sin \theta}{\sin \theta}$$

$$\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \cdot \frac{(\cos \theta + \sin \theta)}{(\cos \theta + \sin \theta)}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta}$$

$$\frac{\cos(2\theta)}{1 + \sin(2\theta)}$$

22. $\frac{1 + \sin \theta}{1 - \sin \theta}$

$$22. \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4 \tan\theta \sec\theta$$

$$1 + 2\sin\theta + \sin^2\theta - 1 + 2\sin\theta - \sin^2\theta$$

$$\frac{4\sin\theta}{1-\sin^2\theta}$$

$$\frac{4\sin\theta}{\cos^2\theta} = \frac{4\sin\theta}{\cos\theta \cos\theta} = 4 \tan\theta \sec\theta$$

$$23. \frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$$

$$\frac{1-\sin x}{\cos x} \cdot \frac{(1+\sin x)}{(1+\sin x)}$$

$$\frac{1-\sin^2 x}{\cos x (1+\sin x)}$$

$$\frac{\cos^2 x}{\cos x (1+\sin x)} = \frac{\cos x}{1+\sin x}$$

$$24. \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$$

omit

$$25. \frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$$

$$\frac{\cos\theta - \sin\theta}{\sin\theta \cos\theta}$$

$$\frac{\cos\theta + \sin\theta}{\sin\theta \cos\theta}$$

$$\frac{\cos^2\theta - \sin^2\theta}{\sin\theta \cos\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos\theta}$$

$$\frac{\cos^2\theta - \sin^2\theta}{1}$$

$$= \cos(2\theta)$$

$$26. \tan\frac{\theta}{2} = \csc\theta - \cot\theta$$

$$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$\frac{1-\cos\theta}{\sin\theta}$$

$$\tan\frac{\theta}{2}$$

$$27. \sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$

$$\sin\left(\frac{3\pi}{2}\right)\cos\theta + \cos\frac{3\pi}{2}\sin\theta$$

$$-1\cos\theta + 0\sin\theta$$

$$= -\cos\theta$$

Review of Law of Sines & Cosines

Date _____

Find each measurement indicated. Round your answers to the nearest tenth.

1) $a = 12$ cm, $b = 21$ cm, $c = 13$ cm
Find $m\angle A$

$$m\angle A \approx 31.4^\circ$$

2) $m\angle A = 113^\circ$, $b = 19$ mi, $c = 17$ mi
Find $m\angle B$

$$m\angle B \approx 35.6^\circ$$

3) $m\angle A = 52^\circ$, $c = 35$ m, $a = 30$ m
Find $m\angle C$

$$\angle C = 66.8^\circ$$
$$\angle C \approx 113.2^\circ$$

4) $m\angle C = 143^\circ$, $b = 6$ in, $c = 26$ in
Find $m\angle B$

$$m\angle B = 8^\circ$$

5) $c = 7$ cm, $m\angle B = 30^\circ$, $a = 19$ cm
Find b

$$b = 13.4 \text{ cm}$$

6) $a = 12$ in, $c = 29$ in, $m\angle B = 51^\circ$
Find b

$$b = 23.4 \text{ cm}$$

7) $m\angle A = 18^\circ$, $c = 34$ km, $a = 23$ km
Find b

$$b \approx 52.8 \text{ km}$$
$$b \approx 11.9 \text{ km}$$

8) $m\angle B = 57^\circ$, $a = 35$ mi, $b = 20$ mi
Find c

not a Δ