Sequences & Series Work

1) $a_1 = \frac{1}{1+1} = 1/2$
$a_2 = \frac{2}{2+1} = 2/3$
$a_3 = \frac{3}{3+1} = 3/4$
$a_4 = 4/4+1 = 4/5$
$a_5 = 5/5+1 = 5/6$

2. $a_1 = 1e+1$
$a_2 = 2e+2$
$a_3 = 3e+3$
$a_4 = 4e+4$
$a_5 = 5e+5$

3. $a_2 = 4(2) = 8$
$a_3 = 4(8) = 32$
$a_4 = 4(32) = 128$
$a_5 = 4(128) = 512$

4. $a_1 = (-1)^{1-1}(2\cdot1) = 2$
$a_2 = (-1)^{2-1}(2\cdot2) = -4$
$a_3 = (-1)^{3-1}(2\cdot3) = 6$
$a_4 = (-1)^{4-1}(2\cdot4) = -8$
$a_5 = (-1)^{5-1}(2\cdot5) = 10$

Typo n-1

5. $a_2 = -2(-2)-7 = -3$
$a_3 = -2(-3)-7 = -1$
$a_4 = -2(-1)-7 = -5$
$a_5 = -2(-5)-7 = 3$

6. $a_1 = \frac{1^2}{1+1} = \frac{1}{2}$
$a_2 = 2^2/_{1+2} = 4/3$
$a_3 = \frac{3^2}{1+3} = 9/4$
$a_4 = \frac{4^2}{4+1} = \frac{16}{5}$
$a_5 = \frac{5^2}{5+1} = \frac{25}{6}$

7. Find a pattern
8. Arith. $d=3$ $a_n = -8+3(n-1)$ or $a_n = 3n-11$
9. Geo $r=-2$ $a_n = 3(-2)^{n-1}$

10. $a_9 = 18$
$a_{53} = 7$
$d = \frac{7-18}{53-9} = -\frac{1}{4}$
$18 = a_1 - \frac{1}{4}(9-1)$
$18 = a_1 - 1/4(9) + 1/4$
$17.75 = a_1 - 4/4$
$20 = a_1$

$\boxed{a_n = 20 - 1/4(n-1)}$

11. $a_{10} = -40$
$a_{82} = 176$
$d = \frac{176 - -40}{82-10} = 3$
$176 = a_1 + 3(82-1)$
$176 = a_1 + 243$
$-67 = a_1$

$\boxed{a_n = -67 + 3(n-1)}$

12. $a_4 = -20$ $\quad a_{10} = -4/3125$ $\quad r = \frac{-4/3125}{-20} = \sqrt[6]{.000064} = .2$ ($10-4=6$)

$\boxed{a_n = -2500(.2)^{n-1}}$

$-20 = a_1(.2)^{4-1}$

$-2500 = a_1$

13. $a_3 = 81$ $\quad a_9 = 1/9$ $\quad r = \frac{1/9}{81} = \sqrt[6]{.00137} = 1/3$ ($9-3=6$)

$\boxed{a_n = 729(1/3)^{n-1}}$

$81 = a_1(1/3)^{3-1}$

$729 = a_1$

14. $10+10+10+10+10 = 50$

15. $S = 6/2(8+18) = \boxed{78}$

$a_2 = 2(2)+4 = 8$ $\quad a_7 = 2(7)+4 = 18$

16. $a_{-2} = (-2)^2+-2 = 2$ $\quad a_{-1} = (-1)^2+-1 = 0$

$a_0 = (0)^2+0 = 0$ $\quad a_1 = 1^2+1 = 2$

$a_2 = 2^2+2 = 6$ $\quad a_3 = 3^2+3 = 12$

Sum $= 2+0+0+2+6+12 = \boxed{22}$

17. Infinite, convergent series $\quad S = \frac{5}{1-2/3} = \boxed{15}$

18. Infinite, convergent series $\quad S = \frac{4/27}{1-1/3} = \boxed{2/9}$

19. No sum, goes on forever

20. $\sum_{n=1}^{28}(3n-2)$ $\quad$ Arith. $d=3$ $\quad a_n = 1+3(n-1)$

$a_n = 1+3n-3$

$a_n = 3n-2$

$82 = 3n-2$

$84 = 3n \quad n = 28$

21. $\sum_{n=1}^{43}(-6n+29)$ Arithm $d=-6$

$a_n = 23-6(n-1)$
$a_n = 23-6n+6$
$a_n = -6n+29$

$-229 = -6n+29$
$-258 = -6n$ $n=43$

22. $\sum_{n=1}^{7} 5(3)^{n-1}$ Geo $r=3$

23. $\sum_{n=1}^{7} 2(1/4)^{n-1}$ Geo $r=1/4$

24. $\sum_{n=1}^{11}(-1)^{n-1}\cdot n^2$ Find a pattern

25. $4047 = \frac{n}{2}(15+a_n)$

$a_n = 15+2(n-1)$
$a_n = 2n+13$

$8094 = n(15+2n+13)$
$8094 = 2n^2+28n$
$0 = 2n^2+28n-8094$
$n = \frac{-28 \pm \sqrt{28^2-4(2)(-8094)}}{2(2)}$

$n=57$

26. $315 = \frac{n}{2}(315+a_n)$

$a_n = 315-9(n-1)$
$a_n = -9n+324$

$315 = \frac{n}{2}(315-9n+324)$
$630 = n(-9n+639)$
$0 = -9n^2+639n-630$
$n = \frac{-639 \pm \sqrt{639^2-4(-9)(-630)}}{2(-9)}$

$n=1$

27. $-32767 = \frac{-1(1-2^n)}{-1+-2}$

$-32767 = 1-2^n$
$-32768 = -2^n$
$\frac{\log(32768)}{\log 2} = \frac{n\log 2}{\log 2}$

$n=15$

28. $531440 = \frac{2(1-3^n)}{-2}$

$-531440 = 1-3^n$

$-531441 = -3^n$

$\frac{\log 531441}{\log 3} = \frac{n \log 3}{\log 3}$

$\boxed{n = 12}$

29. 16, 48, 80, ...

Arith $d = 32$

$a_n = 16 + 32(n-1)$

$a_n = 16 + 32(6-1)$

$a_n = 176$

$S = \frac{6}{2}(16+176)$

$\boxed{S = 576 \text{ total feet}}$

30. 180, 360, 540, ...

Arith $d = 180$

$a_n = 180 + 180(n-3)$

$a_n = 180 + 180(50-3)$

$\boxed{a_n = 8640°}$

31. 500, 530, ...

Geo $r = 1.06$

$a_n = 500(1.06)^{n/2}$ → divide by 2 since it doubles every 2 hrs

$a_n = 500(1.06)^{24/2} = \boxed{10006.098 \text{ hrs}}$

32. No question... but the equation is

$a_n = 2500(.55)^{n-1}$

↳ 1 − .45

33. 2000, 1600, 1280, ...

Geo $r = 4/5 \rightarrow 1 - 1/5$

$a_n = 2000(4/5)^{n-1}$ $\boxed{a_5 = 655.36 \text{ lbs}}$

34. $(2x^3-3)^4$

$= (2x^3)^4 + 4(2x^3)^3(-3) + 6(2x^3)^2(-3)^2 + 4(2x^3)^1(-3)^3 + (-3)^4$

$= 16x^{12} - 96x^9 + 216x^6 - 216x^3 + 81$

35. $(3x^3+4)^6$

$= (3x^3)^6 + 6(3x^3)^5(4) + 15(3x^3)^4(4)^2 + 20(3x^3)^3(4)^3 + 15(3x^3)^2(4)^4 + 6(3x^3)(4)^5 + (4)^6$

$= 729x^{18} + 5832x^{15} + 19440x^{12} + 34560x^9 + 34560x^6 + 18432x^3 + 4096$

36. $(y+2)^9$

$84(y)^3(2)^6 = 5376$

37. see above

38. $(2a-3b)^{10}$

$210(2a)^4(-3b)^6 = 2449440a^4b^6$

39. $(x^2+4y)^9$

$126(x^2)^5(4y)^4 = 32256x^{10}y^4$