

Name: Key

Solving Trig Equations - More Complicated Ones

Solve each equation for $0 \leq \theta < 2\pi$.

1) $\tan\left(\theta + \frac{\pi}{6}\right) = \sqrt{3}$ $\frac{\pi}{6}, \frac{7\pi}{6}$

$\theta + \frac{\pi}{6} = \frac{\pi}{3} + \pi K$ $\theta + \frac{\pi}{6} = \frac{4\pi}{3} + \pi K$
 $\theta = \frac{\pi}{6} + \pi K$ $\theta = \frac{7\pi}{6} + \pi K$

2) $\cos(-4\theta) = 1$

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$-4\theta = 0 + 2\pi K$
 $\theta = 0 + \frac{\pi}{2} K$

3) $\sin\left(\theta + \frac{\pi}{3}\right) = 0$ $\frac{5\pi}{3}, \frac{2\pi}{3}$

$\theta + \frac{\pi}{3} = 0 + 2\pi K$ $\theta + \frac{\pi}{3} = \pi + 2\pi K$
 $\theta = -\frac{\pi}{3} + 2\pi K$ $\theta = \frac{2\pi}{3} + 2\pi K$

4) $-\frac{\sqrt{3}}{2} = \cos 2\theta$

$\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$

$2\theta = \frac{5\pi}{6} + 2\pi K$ $2\theta = \frac{7\pi}{6} + 2\pi K$
 $\theta = \frac{5\pi}{12} + \pi K$ $\theta = \frac{7\pi}{12} + \pi K$

5) $\sin \frac{\theta}{3} = \frac{1}{2}$ $\frac{\pi}{2}$

$\frac{\theta}{3} = \frac{\pi}{6} + 2\pi K$ $\frac{\theta}{3} = \frac{5\pi}{6} + 2\pi K$
 $\theta = \frac{\pi}{2} + 6\pi K$ $\theta = \frac{5\pi}{2} + 6\pi K$

6) $\sin(-4\theta + \pi) = 0$

$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

7) $1 = \sin(2\theta + \pi)$

$\frac{3\pi}{4}, \frac{7\pi}{4}$

$\sin\left(2\left(\frac{3\pi}{4}\right) + \pi\right)$

8) $\tan\left(\pi + \frac{\theta}{3}\right) = 1$

$\frac{3\pi}{4}$

* Check on Calculator

Factor to solve each of the following. Use the unit circle to get exact solutions whenever possible. Find ALL solutions in radians. Round decimal answers to the nearest thousandth when needed.

9. $2 \cos^2 x + \cos x = 0$

$\cos \theta (2 \cos \theta + 1) = 0$

$\cos \theta = 0$ $\cos \theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2} + 2\pi K$ $\theta = \frac{2\pi}{3} + 2\pi K$
 $\theta = \frac{4\pi}{3} + 2\pi K$

10. $2 \sin^2 x - \sin x - 1 = 0$

$(2 \sin \theta + 1)(\sin \theta - 1) = 0$

$\sin \theta = -\frac{1}{2}$ $\sin \theta = 1$

$\theta = \frac{7\pi}{6} + 2\pi K$ $\theta = \frac{\pi}{2} + 2\pi K$
 $\frac{11\pi}{6} + 2\pi K$

$$11. 3 \sin^2 x + 2 \sin x = 5$$

$$3 \sin^2 \theta + 2 \sin \theta - 5 = 0$$

$$(3 \sin \theta + 5)(\sin \theta - 1) = 0$$

$$\sin \theta = -5/3 \quad \sin \theta = 1$$

$$\theta = \pi/2 + 2\pi k$$

$$13. 1 - \cos^2 x = 1 + \cos x + \cos^2 x$$

$$0 = \cos^2 x + 2 \cos x$$

$$0 = \cos x (1 + 2 \cos x)$$

$$\cos x = 0 \quad \cos x = -1/2$$

$$15. \cos^3 x = \cos x$$

$$\cos^3 x - \cos x = 0$$

$$\cos x (\cos^2 x - 1) = 0$$

$$\cos x (\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x = -1 \quad \cos x = 1$$

$$\pi/2 + 2\pi k \quad \pi + 2\pi k \quad 0 + 2\pi k$$

$$3\pi/2 + 2\pi k$$

$$17. 25 \sin x \cos x - 5 \sin x + 20 \cos x = 4$$

$$25 \sin x \cos x - 5 \sin x + 20 \cos x - 4 = 0$$

$$5 \sin x (5 \cos x - 1) + 4(5 \cos x - 1) = 0$$

$$(5 \sin x + 4)(5 \cos x - 1) = 0$$

$$\sin x = -4/5 \quad \cos x = 1/5$$

$$x = .927 + 2\pi k \quad x = 1.369 + 2\pi k$$

$$x = .927 + \pi \approx 4.069 + 2\pi k$$

$$2\pi - .927 \approx 5.356 + 2\pi k$$

$$12. 2 \tan^2 x + 5 \tan x + 3 = 0$$

$$(\tan x + 1)(2 \tan x + 3) = 0$$

$$\tan x = 1 \quad \tan x = -3/2$$

$$\theta = 3\pi/4 + \pi k \quad \theta = -.983 + \pi k$$

$$\theta = 7\pi/4 + \pi k \quad \theta = 5.3 + \pi k$$

$$14. \cos x \csc x = 2 \cos x$$

$$\cos x \csc x - 2 \cos x = 0$$

$$\cos x (\csc x - 2) = 0$$

$$\cos x = 0 \quad \csc x = 2$$

$$\sin x = 1/2$$

$$16. 4 \sin^2 x + 7 \sin x = 2$$

$$4 \sin^2 x + 7 \sin x - 2 = 0$$

$$(4 \sin x - 1)(\sin x + 2) = 0$$

$$\sin x = 1/4 \quad \sin x = -2$$

$$x = 0.253 + 2\pi k$$

$$x = 2.889 + 2\pi k$$

$$18. 4 \sin x \tan x - 3 \tan x + 20 \sin x - 1 = 0$$

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