

$$1.) \quad 2\sin\theta + 3 = 2$$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \frac{7\pi}{6}, \theta = \frac{11\pi}{6}$$

$$2.) \quad 2\sin^2\theta - 1 = 0$$

$$2\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \pm\frac{\sqrt{2}}{2}$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\sin\theta = -\frac{\sqrt{2}}{2}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$3.) \quad 4\cos^2\theta - 3 = 0$$

$$4\cos^2\theta = 3$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\sqrt{\frac{3}{4}}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \quad \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$4.) \quad \tan\theta + 1 = 0$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$5.) \quad \sqrt{3}\cot\theta + 1 = 0$$

$$\sqrt{3}\cot\theta = -1$$

$$\cot\theta = -\frac{1}{\sqrt{3}}$$

$$\tan\theta = -\sqrt{3}$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$6.) \quad 4\sec\theta + 6 = -2$$

$$4\sec\theta = -8$$

$$\sec\theta = -2$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$7.) \quad 2\cos^2\theta + \cos\theta = 0$$

$$\text{GCF: } \cos\theta(2\cos\theta + 1) = 0$$

$$\cos\theta = 0 \quad 2\cos\theta + 1 = 0$$

$$\theta = \cos^{-1}(0) \quad 2\cos\theta = -1$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$8.) \quad 2\sin^2\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$2\sin\theta = -1 \quad \sin\theta = 1$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \sin^{-1}(1)$$

or let  $u = \sin\theta$

$$2u^2 - u - 1 = 0$$

$$(2u + 1)(u - 1) = 0$$

$$u = -\frac{1}{2} \quad u = 1$$

$$\sin\theta = -\frac{1}{2} \quad \sin\theta = 1$$

9.)  $(\tan\theta - 1)(\sec\theta - 1) = 0$

$\tan\theta - 1 = 0$      $\sec\theta - 1 = 0$

$\tan\theta = 1$      $\cos\theta = 1$

$\theta = \tan^{-1}(1)$      $\theta = \cos^{-1}(1)$

$\theta = \pi/4, 5\pi/4, 0$

13.)  $2(1 - \cos^2\theta) = 3(1 - \cos\theta)$

$2 - 2\cos^2\theta = 3 - 3\cos\theta$

$0 = 2\cos^2\theta - 3\cos\theta + 1$

$0 = (\cos\theta - 1)(2\cos\theta - 1)$

$\cos\theta = 1$      $\cos\theta = 1/2$

10.)  $1 - \cos^2\theta = 1 + \cos\theta + \cos^2\theta$   
 $+ \cos^2\theta$      $+ \cos^2\theta$

$1 = 1 + \cos\theta + 2\cos^2\theta$

$0 = \cos\theta + 2\cos^2\theta$

GCF

$0 = \cos\theta(1 + 2\cos\theta)$

$\cos\theta = 0$      $1 + 2\cos\theta = 0$

$\cos\theta = -1/2$

$\theta = \pi/2, 3\pi/2, 2\pi/3, 4\pi/3$

11.)  $1 - \sin^2\theta + \sin\theta = \sin^2\theta$   
 $+ \sin^2\theta$      $+ \sin^2\theta$

set = 0

$1 + \sin\theta = 2\sin^2\theta$

$0 = 2\sin^2\theta - \sin\theta - 1$

factor

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$2\sin\theta + 1 = 0$      $\sin\theta - 1 = 0$

$\sin\theta = -1/2$      $\sin\theta = 1$

$\theta = \sin^{-1}(-1/2)$      $\theta = \sin^{-1}(1)$

$\theta = 7\pi/6, 11\pi/6$

(\*15)  $\sec^2\theta - 1 = 3 \cdot \sec\theta$

$\sec^2\theta - 3 \cdot \sec\theta - 1 = 0$

$2\sec^2\theta - 3\sec\theta - 2 = 0$

$(2\sec\theta + 1)(\sec\theta - 2) = 0$

$\sec\theta = -1/2$      $\sec\theta = 2$

$\cos\theta = -2$      $\cos\theta = 1/2$

$\theta = \pi/3, 5\pi/3$

12.)  $1 - \cos^2\theta = 6(\cos\theta + 1)$

$1 - \cos^2\theta = 6\cos\theta + 6$

$0 = \cos^2\theta + 6\cos\theta + 5$

$0 = (\cos\theta + 5)(\cos\theta + 1)$

$\cos\theta = -5$      $\cos\theta = -1$

$\theta = \pi$

16.)  $\cos^3x = \cos x$

$\cos^3x - \cos x = 0$

diff of perfect sq

$\cos x(\cos^2x - 1) = 0$

$\cos x(\cos x + 1)(\cos x - 1) = 0$

$\cos x = 0$      $\cos x + 1 = 0$      $\cos x - 1 = 0$

$\cos x = 0$      $\cos x = -1$      $\cos x = 1$

$\theta = \pi/2, 3\pi/2, \pi, 0$