More with Logs

Endpages have work

Example 1) If $\log_7 6 = x$ and $\log_7 5 = y$, express the following in terms of x and y:

a. $\log_7 35$ $\log_7 6$

b. $\log_7(7.2)$

Omit

NO Calculator

Example 2) Evaluate each. a. $log_4 4^x$ $=x$ b. $log_7 7^8$ $=8$ c. $6^{log_6 9}$ $=9$ d. $3^{log_3 17}$ $=17$

Inverse Properties of Logarithms

For $b > 0$, $log_b b^x =$ ____, the logarithm with base b of b raised to a power equals that power.

$b^{log_b x} =$ ____, b raised to the logarithm with base b of a number equals that number.

PRACTICE

For numbers 1 – 6, let $a = \log_{11} 3$, and $b = \log_{11} 10$. Express the following in terms of a and b:

Omit

____ 1) $\log_{11} 300$ ____ 2) $\log_{11} 90$ ____ 3) $\log_{11} 0.3$

____ 4) $\log_{11}\left(\frac{100}{3}\right)$ ____ 5) $\log_{11} 900$ ____ 6) $\log_{11} 33$

Evaluate each expression without a calculator.

7. $\log_7 7^3$ $=3$

8. $\log 0.001$ $\log\frac{1}{1000} = \log 10^{-3} = -3$

9. $3^{\log_3 6}$ $=6$

10. $4^{2\log_4 5}$ $=25$

11. $\log_5 1$ $=0$

12. $\log_6 4 + 2\log_6 3$ $\log_6 4 \cdot 3^2 = \log_6 36 = \log_6 6^2 = 2$

13. $\frac{1}{2}\log_3 144 - 2\log_3 6$ $= \log_3\sqrt{144} - \log_3 6^2 = \log_3\left(\frac{12}{36}\right) = \log_3\frac{1}{3} = -1$

14. lne^5 $=5$

15. $ln\frac{1}{e}$ $=-1$

16. $ln\sqrt[7]{e^4}$ $= lne^{4/7} = 4/7$

17. $\ln 0$ DNE

18. $e^{ln6+ln5}$ $e^{ln30} = 30$

19. $e^{\frac{1}{3}ln8-\frac{1}{2}ln9}$ $e^{ln\sqrt[3]{8} - ln\sqrt{9}}$ $e^{ln 2/3} = 2/3$

20. $\frac{1}{2}ln4 + ln8 - (5ln2 + ln3)$ $ln\frac{\sqrt{4}\cdot 8}{2^5 \cdot 3} = ln\frac{16}{96} = ln$

21. $e^x = 5$ $ln(5) = x$

22. $e^{lnx} = 12$ $x = 12$

23. $x = lne^{\frac{3}{5}}$ $x = 3/5$

Honors Math 3 - Logarithm Unit Review

Write as a single logarithm:

* 1. $\log 3x + 3\log y - 2\log(wz)$ $\log \frac{3xy^3}{(wz)^2}$

2. $\ln \frac{\sqrt{x}}{x} + \ln \sqrt[4]{ex^2}$ $\ln\left(\frac{\sqrt{x}}{x} \cdot \sqrt[4]{ex^2}\right)$

Expand the following:

3. $\log_4 x^2y^4$ $2\log x + 4\log y$

4. $\ln \frac{\sqrt[3]{x}}{m^3}$ $\frac{1}{3}\ln x - 3\ln m$

* ***Solve the equation:*** Can use calculator – see end pages

5. $\left(\frac{1}{4}\right)^{x+2} = 4^{2x-3}$ $4^{-1(x+2)} = 4^{2x-3}$

6. $3^{x-4} = 4^{x-3}$

7. $6e^{2x} = 11$

8. $\log(x - 16) = 2 - \log(x - 1)$

9. $\log(3x - 5) = \log 11 + \log 2$

10. $\ln(x + 9) - \ln x = 1$

11. $\log \sqrt[3]{x^2 + 21x} = \frac{2}{3}$

12. $\ln(x^2 + 1) - \ln(x - 1) = 1 + \ln(x + 1)$

13. $\log^2 x - \log x^9 = 22$ Omit

14. $e^{2x} - e^x - 30 = 0$ Omit

Find the inverse of the following functions:

15. $f(x) = 3 \cdot 2^{x+3}$

16. $f(x) = \sqrt{3^{2x-8}}$

17. $f(x) = \log(x - 6)$

18. $f(x) = 4\ln(x + 2)$

5

Write an equation and solve:

19. Elmer invested \$4000 in an account paying 3 ¼ % interest, compounded continuously. How long will it take for him until there is \$6000 in the account?

20. If money is invested at 5%, compounded monthly, how long will it take for the money to triple?

21. A certain strain of bacteria grows according to the function $f(x) = 5000e^{0.4055x}$, where the time, x, is measured in hours.
   a. What will the population be in 8 hours?
   b. When will the population be 1 million?

22. If an investment of \$2000 grown to \$2700 in 3 ½ years, with an annual interest rate that is compounded quarterly, what is the annual interest rate?

Omit

23. The half-life of Barberium is 140 days. Find the function that gives the amount of barberium remaining from an initial 20 milligrams after t days. Then determine the amount of barberium left after 15 weeks. Find how long it will take for the 20 milligrams to decay to 4 milligrams.

24. Find a model that best fits the following data in homeschooled children:

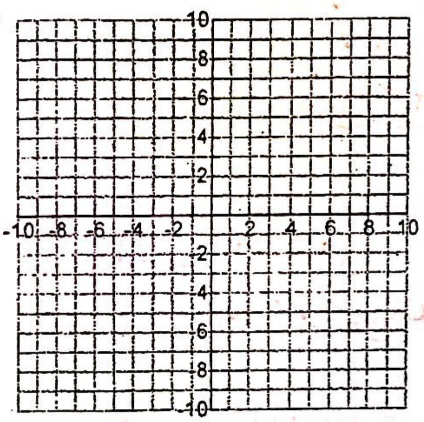
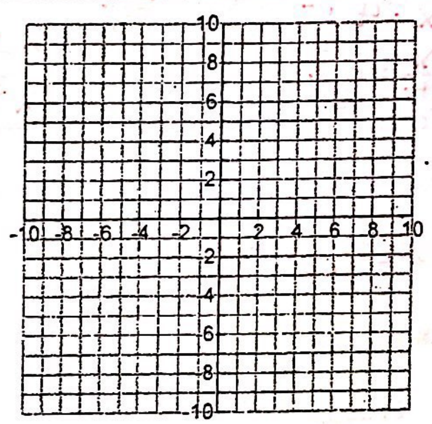
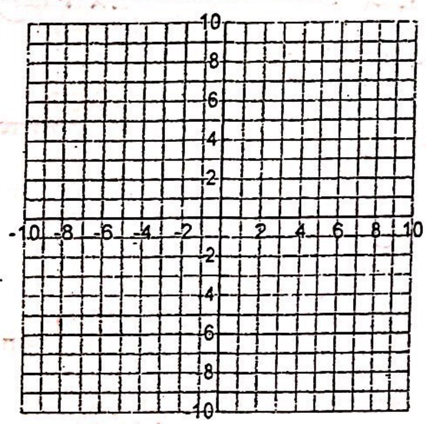
Fall of school year	1985	1988	1990	1992	1993	1994	1995	1996	1997	1999
Number of children (in 1000s)	183	225	301	470	588	735	800	920	1100	1400

What is the number of homeschooled children predicted by the model in 2003?

State the domain

Review Sheet Date: ______________ Period: ____

1. For each of the following functions, state the equation of the base function, the transformations from the base function, the domain, range, asymptotes. If the function is exponential, determine if it is a growth or decay model. Then graph each function.

a. $y = \left(\frac{1}{2}\right)^{x+2} + 3$	b. $y = (2)^{x-1} - 4$	c. $y = -log_2(x-4) + 5$
Base Function:	Base Function:	Base Function:
Transformations: no	Transformations: Graphing	Transformations:
Domain:	Domain:	Domain:
Range:	Range:	Range:
Asymptote:	Asymptote:	Asymptote:
Circle One: **Growth** or **Decay**	Circle One: **Growth** or **Decay**	
		

2. Use the rules of exponents and/or logarithms to find the value of x in each equation. ~~Round to the nearest hundredth when necessary.~~

a. $(3^{2x})(3^{12}) = 3^{20}$

$3^{2x+12} = 3^{20}$

$2x+12 = 20$ $x=4$

b. $\frac{5^8}{5^{2x}} = 5^{10}$ $x=-1$

$5^{8-2x} = 10$

$8-2x = 10$

c. $(13^4)^x = 13^{24}$

$4x = 24$

$x = 6$

d. $(25^{2x})(5^7) = 125^4$

$5^{2(2x)} \cdot 5^7 = 5^{3(4)}$ $x = 5/4$

$4x + 7 = 12$

e. $\frac{9^{5x}}{3^{2x}} = 81^{12}$

$\frac{3^{2(5x)}}{3^{2x}} = 3^{4(12)}$ $x=6$

$3^{10x-2x} = 3^{48}$

f. $(8^4)^x = 4^{18}$

$2^{3(4x)} = 2^{2(18)}$ $x=3$

$12x = 36$

g. $(49^{2x})(7^8) = 1$ $x=-2$

$(7^2)^{2x} \cdot 7^8$

$7^{4x+8} = 0$

$4x = -8$

h. $(25)^{\frac{1}{2}}(3)^4 = x$

$5 \cdot 81 = x$

$x = 405$

i. $\left(6^{\frac{1}{2}}\right)\left(36^{\frac{3}{2}}\right) = 6^x$

$6^{1/2} \cdot 6^{2(3/2)} = 6^x$

$6^{1/2} \cdot 6^3 = 6^x$ $3\frac{1}{2} = x$

j. $10^{x+4} = 100{,}000{,}000$

$10^{x+4} = 10^8$

$x+4 = 8$

$x = 4$

k. $\frac{6(10)^{5x}}{6} = \frac{18{,}000}{6}$

$10^{5x} = 3{,}000$

$\log_{10}(3000) = 5x$

$= .6954$

l. $10^{3x-4} = 1{,}000$

$10^{3x-4} = 10^3$

$3x - 4 = 3$

$3x = 7$

$x = 7/3$

33

p. $-5(10)^{x-9} = -5{,}000$ $\frac{}{5}$ $\frac{}{-5}$ x=12

$10^{x-9} = 10^3$

$x-9=3$

q. $\frac{2}{1}\cdot\frac{1}{2}(10)^{2x} = 50{,}000 \cdot \frac{2}{1}$ $x=\frac{5}{2}$

$10^{2x} = 100{,}000$

$10^{2x} = 10^5$

r. $10^{2x} = .0001$

$10^{2x} = \frac{1}{10{,}000}$

$10^{2x} = 10^{-4}$ x=-2

s. $\log(x+5) = 2$

$10^2 = x+5$

$95 = x$

t. $\log_3(4x-3) = 4$

$3^4 = 4x-3$

$84 = 4x$ x=21

u. $\log_x 8 = 3$

$x^3 = 8$

$2^3 = x^3$ x=2

v. $\log_x 144 = 2$

$x^2 = 144$

$x^2 = 12^2$ x=12

w. $\log_4(4x) = 3$

$4^3 = 4x$

$64 = 4x$ x=16

x. $\log(25x) = 2$

$10^2 = 25x$

$4 = x$

3. Find the inverse of each function

a. $y = \frac{1}{2}x - 5$ $y^{-1} = 2x+10$

$x = \frac{1}{2}y - 5$

$x+5 = \frac{1}{2}y$

b. $y = 4x^2$ $y^{-1} = \frac{\pm\sqrt{x}}{2}$

$x = 4y^2$

$\frac{x}{4} = y^2$

c. $y = \sqrt[3]{x+4}$

$x = \sqrt[3]{y+4}$

$x^3 - 4 = y^{-1}$

4. Rewrite each function in ~~exponential~~ log form. (2 points each)

Omit section

a. $216 = 6^x$

$6^3 = 6^x$

$x = 3$

b. $x = 12^6$

$\log_{12} x = 6$

c. $81 = 3^{8x}$

$3^4 = 3^{8x}$ $x = \frac{1}{2}$

$\log_3 81 = 8x$

5. Rewrite each function in ~~logarithmic~~ Exp form. (2 points each)

a. $log_3 243 = x$ $x=5$

$3^x = 243$

$3^x = 3^5$

b. $log_{15} x = 3$

$15^3 = x$

d. $log_x 120 = 3$

$x^3 = 120$

6. Suppose that 500 mg of a medicine enters a hospital patient's bloodstream at noon and decays exponentially at a rate of 15% per hour. The exponential function $D(t) = 500(10^{-0.07t})$ models the amount of medicine active in the patient's blood at a time t hours later, where t is time in hours. Round answers to the nearest hundredth.

$y = P(1-r)^t$

a. Find D(0). 500 mg

b. Find D(3). $500(10^{-.07(3)})$ ≈ 308.3 mg

c. Use logarithms to determine when there is 150 mg of medicine in the patient's blood stream.

$150 = 500(10^{-.07t})$

$\frac{150}{500} = 10^{-.07t}$ $\rightarrow$ $\log\left(\frac{150}{500}\right) = -.07t$

$t = 7.5$ hr

d. Use logarithms to determine when there is 10 mg of medicine in the patient's blood stream.

$10 = 500(10^{-.07t})$

$\log(10/500) = -.07t$ $t = 24.3$ hr

7. The function $y = 12{,}800\,(1.045)^x$ represents the value of a piece of artwork x years after purchase.

a. How much will the artwork be worth in 15 years?

$y = 12{,}800(1.045)^{15}$

$y = \$24{,}771.62$

34

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$x\ 8 = 3$ $\quad -3 \quad y \neq 3$ $\quad$ Range: $(-\infty, \infty)$

5) $4^{-1(x+2)} = 4^{2x-3}$

$-x-2 = 2x-3$

$1 = 3x$

$1/3 = x$

6) $\log 3^{x-4} = \log 4^{x-3}$

$(x-4)\log 3 = (x-3)\log 4$

$x\log 3 - 4\log 3 = x\log 4 - 3\log 4$

$x\log 3 - x\log 4 = 4\log 3 - 3\log 4$

$x(\log 3 - \log 4) = 4\log 3 - 3\log 4$

$x = \dfrac{4\log 3 - 3\log 4}{\log 3 - \log 4} = -.8188$

7.) $6e^{2x} = 11$

$e^{2x} = 11/6$

$\ln(11/6) = 2x$

$\dfrac{\ln(11/6)}{2} = x$

$x = .3031$

8) $\log(x-16) + \log(x-1) = 2$

$\log(x-16)(x-1) = 2$

$10^2 = x^2 - 17x + 16$

$0 = x^2 - 17x - 84 = (x-21)(x+4) = 0$

$x = 21 \quad \cancel{x = -4}$

9) $\log(3x-5) = \log 22$

$3x - 5 = 22$

$3x = 27$

$x = 9$

10) $\ln\left(\frac{x+9}{x}\right) = 1$

$e^1 = \frac{x+9}{x}$

$xe^1 = x + 9$

$xe^1 - x = 9$

$x(e-1) = 9$

$x = 9/e-1 = 5.2378$

11) $10^{2/3} = \sqrt[3]{x^2 - 21x}$

$10^{2/3} = (x^2 - 21x)^{1/3}$

$100 = x^2 - 21x$

$x^2 - 21x - 100$

$0 = (x-25)(x+4)$

$x = 25 \quad x = -4$

12) $\ln\left(\frac{x^2+1}{x-1}\right) = 1 + \ln(x+1)$

$\ln\left(\frac{x^2+1}{x-1}\right) - \ln(x+1) = 1$

$\ln\left(\frac{x^2+1}{(x-1)(x+1)}\right) = 1$

$e^1 = \frac{x^2+1}{x^2-1}$

$(x^2-1)e = x^2 + 1$

$x^2e - e = x^2 + 1$

$x^2e - x^2 = 1 + e$

$x^2(e-1) = 1 + e$

$x^2 = \dfrac{1+e}{e-1} = 1.471$

$x = \cancel{\pm}\sqrt{\frac{1+e}{e-1}}$ just (+)

13) $\log\left(\frac{x^2}{x^9}\right) = 22$

$\log\left(\frac{1}{x^7}\right) = 22$

$10^{22} = 1/x^7$

$x^7 = 1/10^{22}$

$x = \sqrt[7]{1/10^{22}}$

omit

14) $e^x(e^2-1) = 30$

$e^x = \frac{30}{e^2-1}$

Omit

$\ln\left(\frac{30}{e^2-1}\right) = x$

$f(x) = \frac{\ldots}{x-3}$ ... ermine the inverse ... n and range ... $(\ldots,3)\cup(3,\infty)$

15.) $x = 3 \cdot 2^{y+3}$

$\frac{x}{3} = 2^{y+3}$

$\log_2(x/3) = y+3$

$\log_2(x/3) - 3 = y$

16) $x = \sqrt{3^{2y-8}}$

$x^2 = 3^{2y-8}$

$\log_3 x^2 = 2y-8$

$\log_3(x)^2 + 8 = 2y$

$\frac{\log_3 x^2 + 8}{2} = y$

17.) $x = \log(y-6)$

$10^x = y-6$

$10^x + 6 = y$

18) $x = 4\ln(y+2)$

$\frac{x}{4} = \ln(y+2)$

$e^{x/4} = y+2$

$e^{x/4} - 2 = y$

19) $y = Pe^{rt}$

$\frac{6000}{4000} = \frac{4000e^{.0325t}}{4000}$

$3/2 = e^{.0325t}$

$\ln(3/2) = .0325t$

$\frac{\ln(3/2)}{.0325} = t$

$t = 12.5\text{ year} \approx 13$

20.) $n = 12$

$3 = (1 + \frac{.05}{12})^{12t}$

$\log_{(1+\frac{.05}{12})} 3 = 12t$

$\frac{\log_{1.004} 3}{12} = t$

$22.018 = t$

23 years

21a) $y = 5000e^{.4055(8)}$

$= 128,180.31$

$= 128,181$

b.) $1,000,000 = 5000e^{.4055}$

$200 = e^{.4055x}$

$\ln(200) = .4055x$

$\frac{\ln(200)}{.4055}$

$t \approx 13.07\ldots$

~ 14 years

22.) $y = P(1+\frac{r}{n})^{nt}$

$\frac{2700}{2000} = \frac{2000}{2000}(1+\frac{r}{4})^{4(3.5)}$

$\frac{27}{20} = (1+r/4)^{14}$

$\sqrt[14]{\frac{27}{20}} = 1 + r/4$

$r = .08667$

$\sim 8.7\%$

23

$y = 20(1/2)^{t/140}$

$*15\text{wks} = 105\text{days}$

$y = 20(1/2)^{105/140}$

$\approx 11.9\text{mg}$

$4 = 20(1/2)^{t/140}$

$\frac{4}{20} = (1/2)^{t/140}$

$\log_{1/2}(4/20) = t/140$

$t = 325$ days

24 omit